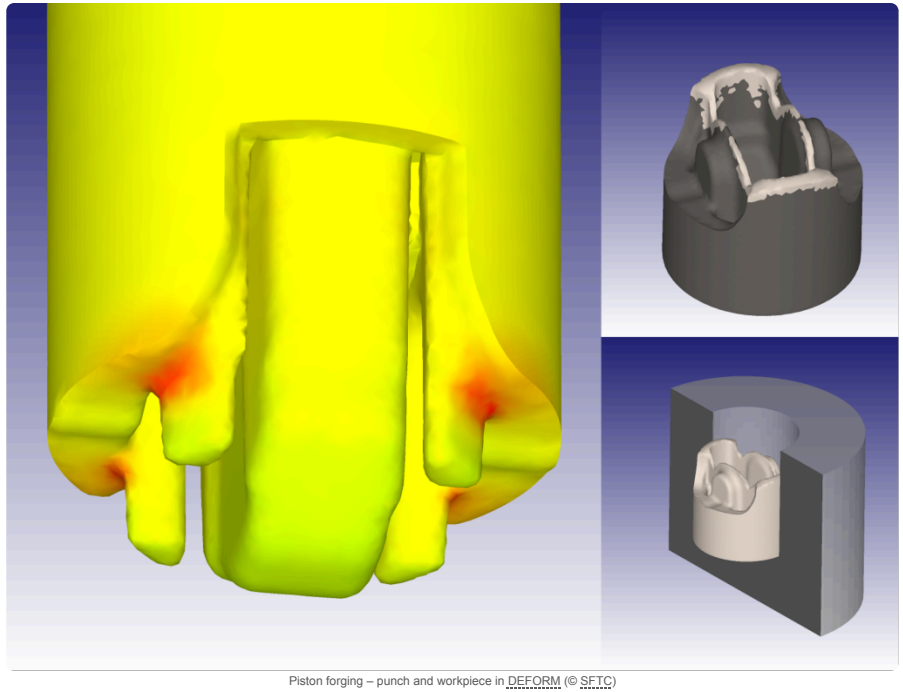
Material Data in DEFORM

Lecture 3 · Modelling and Simulation of Processes I – Forming (MSPO2_6I)

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MATERIÁLOVOTECHNOLOGICKÁ FAKULTA · STU BRATISLAVA



Piston forging – punch and workpiece in DEFORM (© SFTC)

Today's agenda (120 min, no break)

Time	Block	Content
0 – 10	1 · Why it matters	Garbage in, garbage out — one part, two flow curves
10 – 20	2 · Material window	Layout, library, Regular vs Mixture, units
20 – 30	3 · Elastic data	E, v, α(T), hyperelastic materials
30 – 60	**4 · Flow stress**	**Testing, tables, power law, Johnson–Cook, Zerilli–Armstrong, sinh-Arrhenius, choosing a model**
60 – 80	5 · Yield & hardening	von Mises, Hill 48, R-values, Barlat; isotropic, kinematic, Yoshida–Uemori
80 – 90	6 · Creep	Creep curve, DEFORM creep models, relaxation
90 – 105	7 · Thermal data	k, heat capacity, emissivity, density, interface HTC
105 – 115	8 · Beyond mechanics	Grain (JMAK), phases, hardness, diffusion, EM, fracture
115 – 120	9 · Wrap-up	Checklist per process, quiz

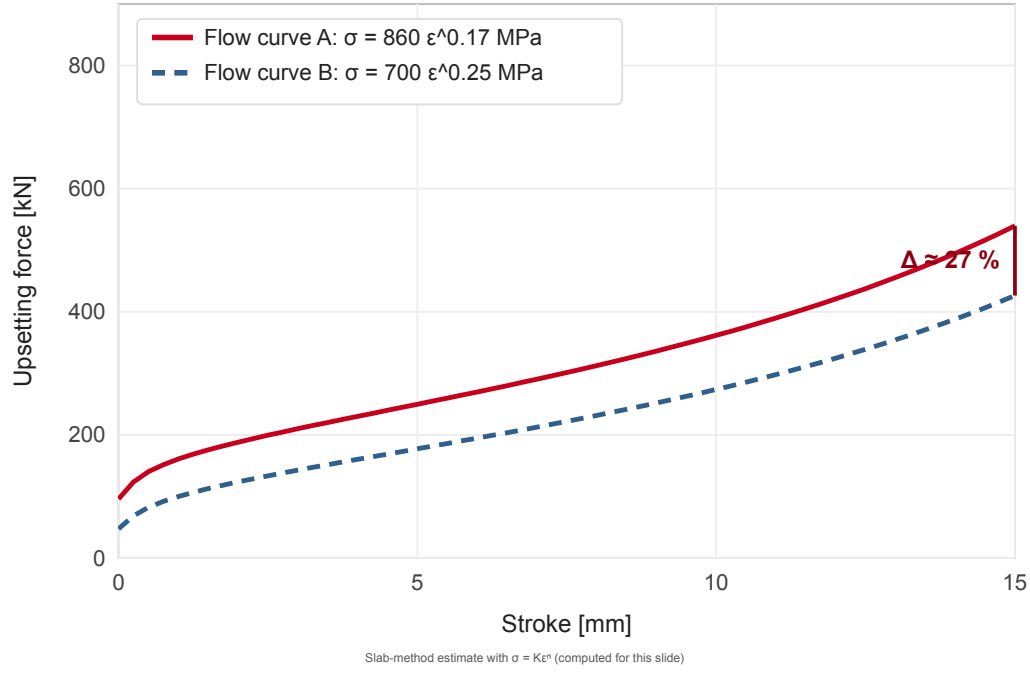
Structure follows DEFORM V14 manual, Chapter 10 “Material Data”

Why it matters Garbage in, garbage out

Same part, same mesh, same friction — two flow curves

- Upsetting Ø20 × 30 mm → 15 mm, μ = 0.1
- **A**: library curve “C45, cold”
- **B**: own test of the “same” grade (different heat, annealed)
- End force: **540 kN** vs **427 kN** — **Δ ≈ 27 %**
- Mesh refinement changes the force by a few % — the **material data** by tens of %

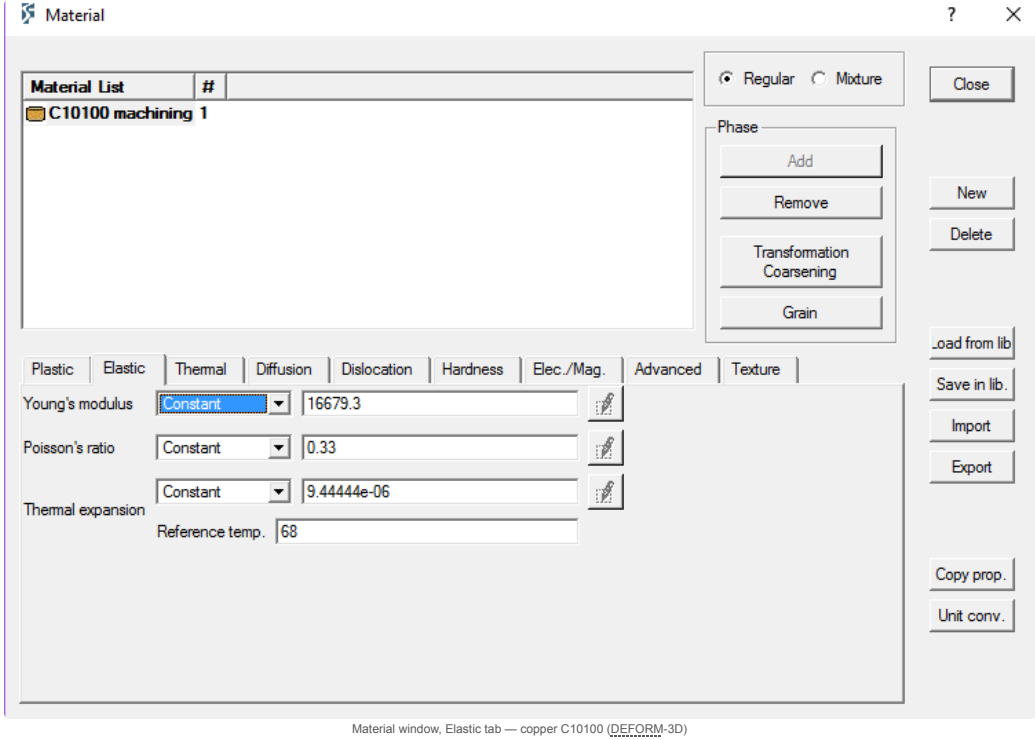
Material data drive: force & energy · temperature · die fill · damage · springback · grain size · hardness



Slab-method estimate with $\sigma = K\varepsilon^n$ (computed for this slide)

Material window Layout

The DEFORM Material window



Material window, Elastic tab — copper C10100 (DEFORM-3D)

- **Material list** — all materials of the project; each object points to one
- **Regular / Mixture** — single phase or several phases
- **Phase** — Add, Remove, Transformation, Grain
- **Tabs**: Plastic · Elastic · Thermal · Diffusion · Dislocation · Hardness · Elec./Mag. · Advanced · Texture
- **Buttons**: New, Delete, Load from lib, Save in lib, Import, Export, Copy prop., **Unit conv.**
- Each property: *Constant* or *f(Temp)*, edited in a table (pencil icon)

Material window Library

Material library: load, save, import, export

Load from lib — choose class (steel, Al, Ti, Ni, Cu, die steels …), grade and temperature range

↓

Material copied *into the project* (.KEY / .DB) **— edits stay local**

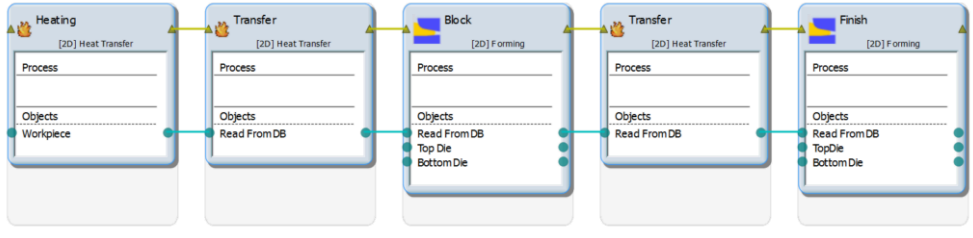
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Save in lib — your own tested data for the next project

↓

Export / Import — material file (.KEY) between users, sites, versions

Library grades carry a **validity range** in the name, e.g. cold 20 °C or hot 900–1200 °C. A “cold” curve in a hot forging is a silent error.



Multiple operations — heating, forging, trimming: one material, many temperatures (© SFTC)

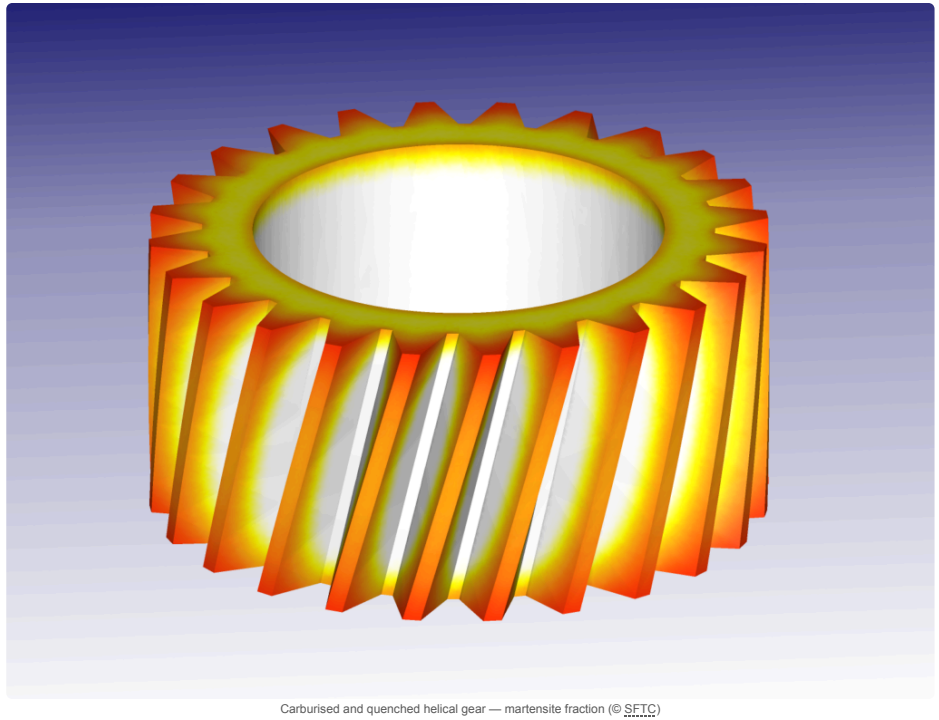
Material window Regular vs Mixture

Regular material or Mixture of phases?

	Regular	Mixture
Phases	one	2 … n (austenite, ferrite, pearlite, bainite, martensite …)
Data	one set per tab	one set *per phase* + transformation data
Use	forming, machining	heat treatment, quenching, carburising

Mixture rule: $P = \sum_i \xi_i P_i, \quad \sum_i \xi_i = 1$

- ξ_i volume fraction of phase i, P_i its property ($\bar{\sigma}$, k, E …)
- **Transformation** button: kinetics (diffusion / martensite), latent heat, volume change, TRIP



Carburised and quenched helical gear — martensite fraction (© SFTC)

Material window Units

Units: the C10100 trap — and which tabs you need

Quantity	English (C10100 above)	SI in DEFORM
E	16 679 ksi	115 000 MPa
α	$9.44 \cdot 10^{-6}$ /°F	$1.7 \cdot 10^{-5}$ /°C
T_{ref}	68 °F	20 °C
Stress	ksi	MPa = N/mm²
k	Btu/(s·in·°F)	N/(s·°C) = W/(m·K)
$\rho \cdot c_p$	Btu/(in³·°F)	N/(mm²·°C) = MJ/(m³·K)
ρ	lb/in³	t/mm³ ($7.7 \cdot 10^{-9}$ for steel)

> Unit system is set per *problem*; mixing an English library entry into an SI problem → use **Unit conv.**

Simulation	Plastic	Elastic	Thermal	Phase / grain
Isothermal forging	●	–	–	–
Hot / warm forging	●	–	●	○
Springback, residual σ	●	●	○	–
Die stress (elastic die)	–	●	○	–
Heat treatment	●	●	●	●
Induction heating	○	○	●	● + EM

● required ○ recommended – not used

Elastic data When needed

When does DEFORM need elastic data?

- **Rigid-plastic** object: elastic strain neglected → **no E, v** needed (most forging)
- **Elastic** object: dies, die inserts, shrink rings → E, v, α
- **Elasto-plastic** object: springback, residual stress, distortion, heat treatment

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^{th} + \varepsilon^{tr}$$

elastic + plastic + thermal + transformation strain

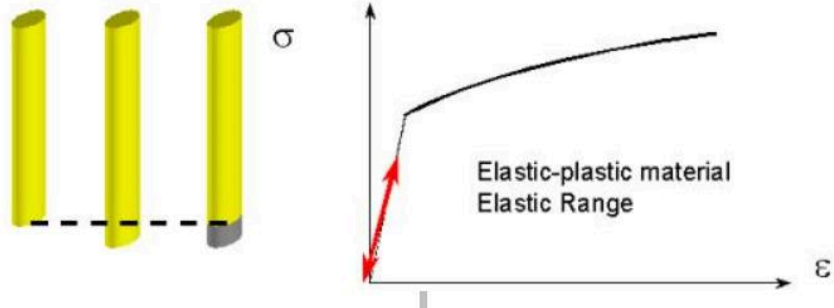
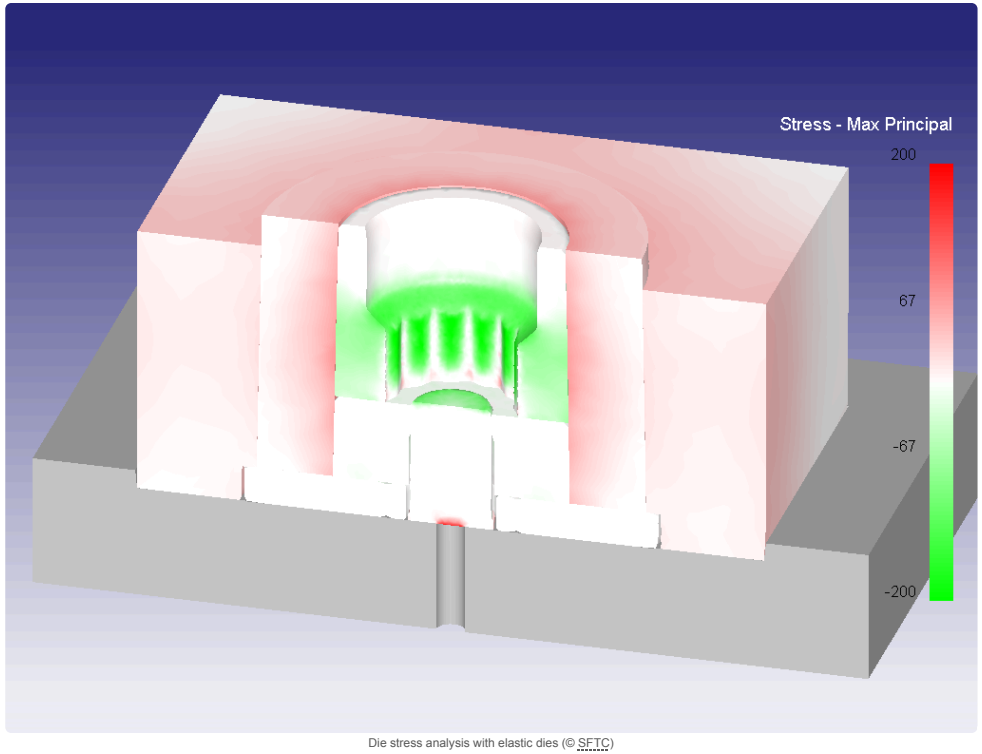


Fig.AI.6. Diagram of elastic deformation

Elastic range: strain recovers fully on unloading



Die stress analysis with elastic dies (© SFTC)

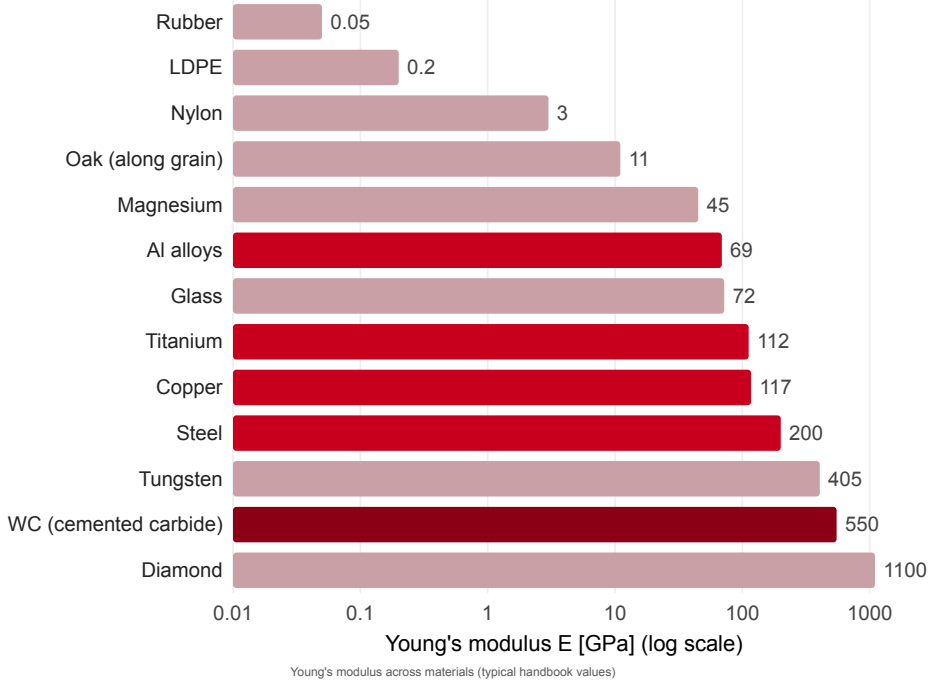
Elastic data E, v

Young's modulus and Poisson's ratio

$$\sigma = E\,\varepsilon^{e}, \qquad \nu = -\frac{\varepsilon_{lat}}{\varepsilon_{ax}}, \qquad G = \frac{E}{2(1+\nu)}$$

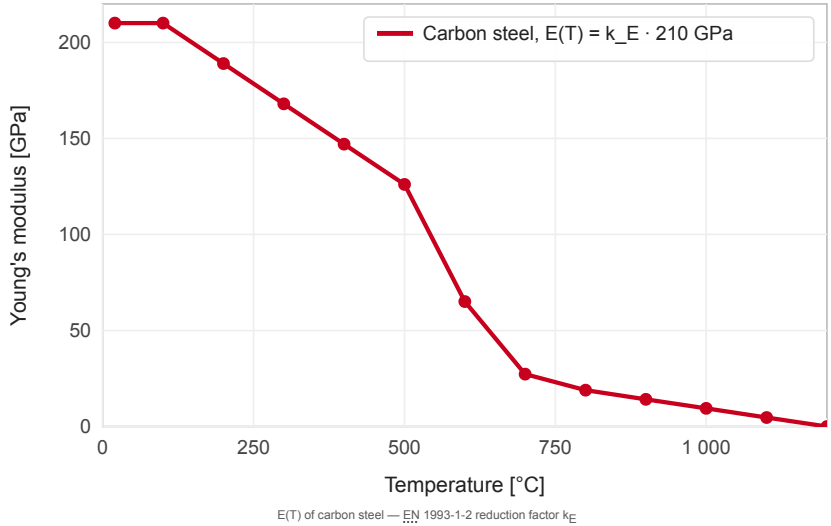
Material	E [GPa]	v
Carbon steel	200 – 210	0.29 – 0.30
Stainless steel (austenitic)	193 – 200	0.29
Aluminium alloys	69 – 72	0.33
Titanium Ti-6Al-4V	110 – 114	0.34
Copper	115 – 120	0.33
WC-Co die insert	500 – 620	0.22

- Springback ∝ σ/E: Al springs back ≈ 3× more than steel at equal stress

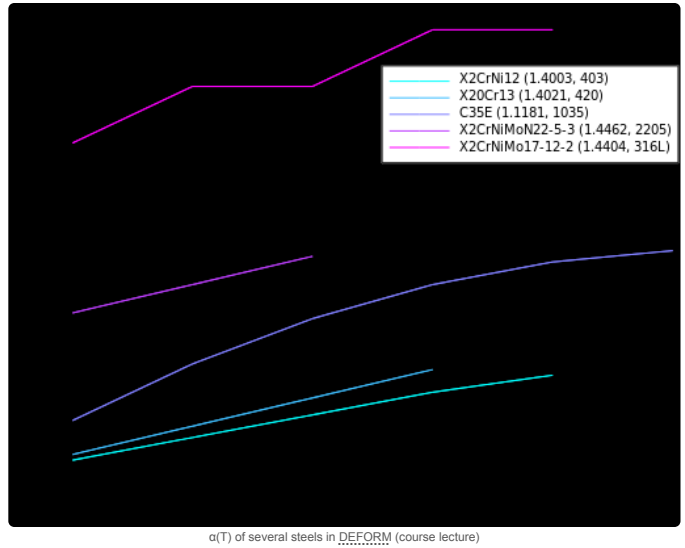


Elastic data Temperature dependence

E(T) and thermal expansion α(T)



E(T) of carbon steel — EN 1993-1-2 reduction factor k_E



α(T) of several steels in DEFORM (course lecture)

$$\varepsilon^{th} = \alpha(T)\,(T - T_{ref}) \qquad \text{DEFORM: “Thermal expansion” + “Reference temp.”}$$

- E of steel falls to about **1/3 at 600 °C** and almost zero above 1000 °C — use *f(Temp)* for quenching and hot die stress
- α is the **secant** value from T_{ref}; austenite expands more than ferrite (≈ 2.2 vs $1.4 \cdot 10^{-5}$ /K)

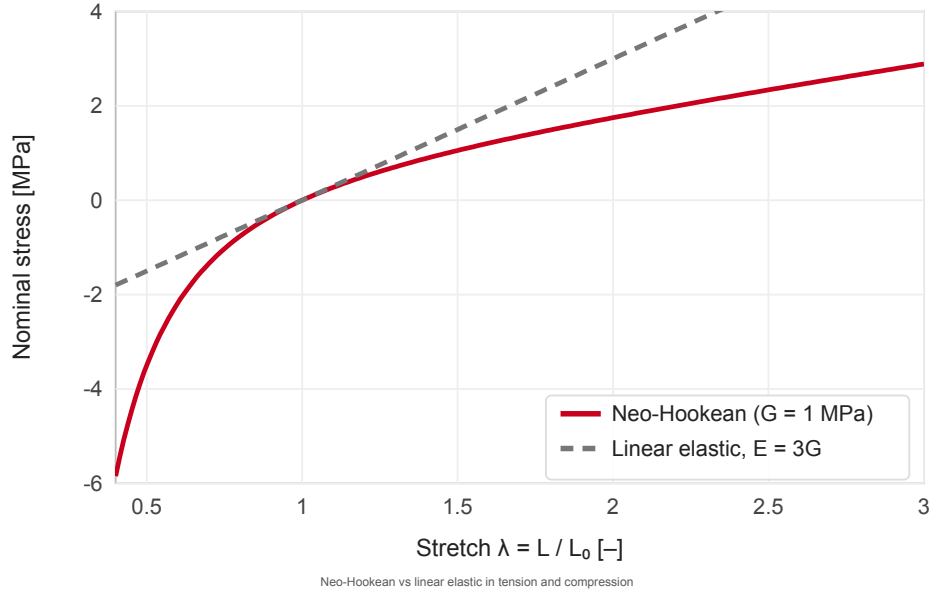
Hyperelastic materials: rubber pads and seals

- Large **reversible** strains (100–500 %), nearly incompressible (v → 0.5)
- Stress from a strain-energy function W instead of E, v

Neo-Hookean: $W = C_{10}(\bar{I}_1 - 3)$
Mooney–Rivlin: $W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3)$

Uniaxial: $\sigma_{nom} = 2C_{10}\,(\lambda - \lambda^{-2}), \quad G = 2C_{10}$

- Forming use: rubber-pad forming, flexible dies, seals in assemblies
- Fit C_{10}, C_{01} from uniaxial + biaxial tests — never from one test only



Neo-Hookean vs linear elastic in tension and compression

Flow stress Definition

Flow stress: $\bar{\sigma} = f(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$

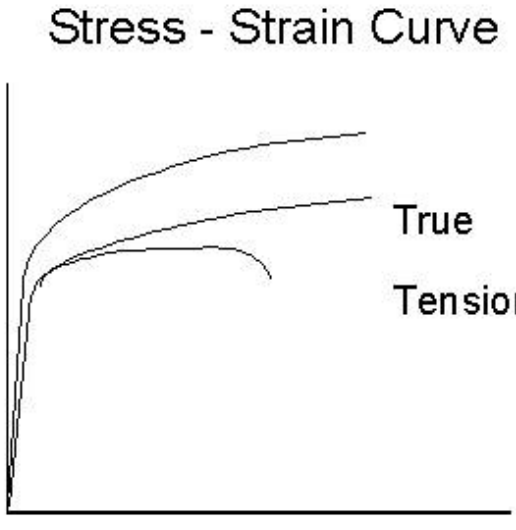


Fig.AI.5. Stress-strain curve

Engineering vs true curve

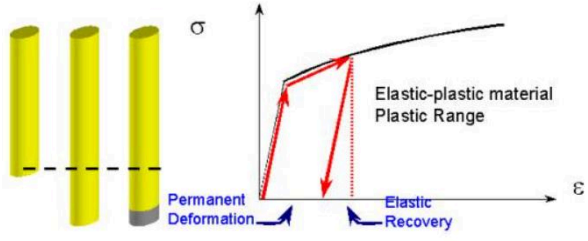


Fig.AI.7. Diagram of plastic deformation

Permanent strain after the yield point

$$\sigma_t = \sigma_e(1+e), \quad \varepsilon = \ln(1+e)$$

- Flow stress = **true** stress needed to keep plastic flow going
- Depends on **strain** (hardening), **strain rate** and **temperature**; also on prior history
- Rigid-plastic FEM uses only the plastic part — the curve starts at the yield stress
- Engineering values are valid only up to necking (uniform elongation)

Flow stress T and ε̇

Temperature softens, strain rate strengthens

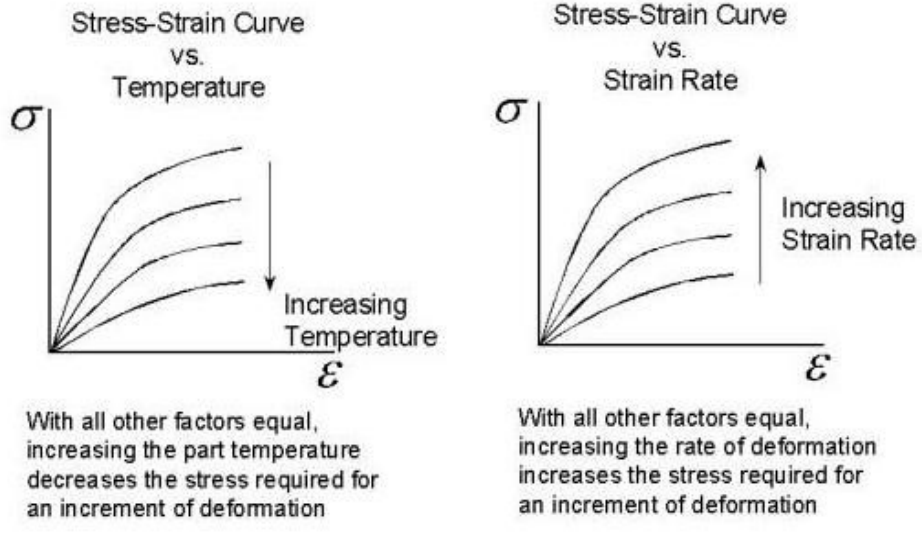


Fig.AI.10. Variation of flow stress with strain rate and temperature

Stress–strain curves vs temperature and strain rate (course lecture)

Regime	T / T_m	Dominant effect
Cold	< 0.3	Strain hardening, weak rate effect (m ≈ 0–0.05)
Warm	0.3 – 0.5	Both, recovery starts
Hot	> 0.5 – 0.6	Rate + T; hardening balanced by recovery / DRX (m ≈ 0.1–0.2)

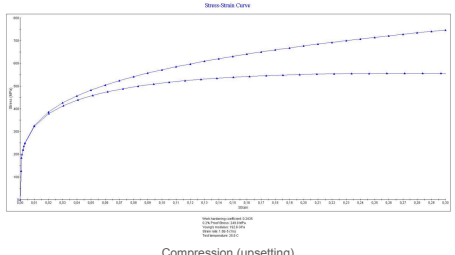
- Strain-rate sensitivity $m = \partial \ln \bar{\sigma} / \partial \ln \dot{\bar{\varepsilon}}$
- Superplastic alloys: m ≥ 0.3–0.5
- Adiabatic heating: $\Delta T \approx \frac{\beta \int \bar{\sigma}\, d\bar{\varepsilon}}{\rho\, c_p}$, β ≈ 0.9

Flow stress Testing

Where flow curves come from



Tensile test
ε < 0.2–0.3, $\dot{\varepsilon} \approx 10^{-3}$–$10^{-1}$ s^{-1} (© ZwickRoell)



Compression (upsetting)
ε ≤ 1, friction correction



Gleeble hot compression
up to ≈ 1700 °C, $\dot{\varepsilon}$ to 10^2 s^{-1} (© DSI)

▶ video: Introduction to Gleeble Systems

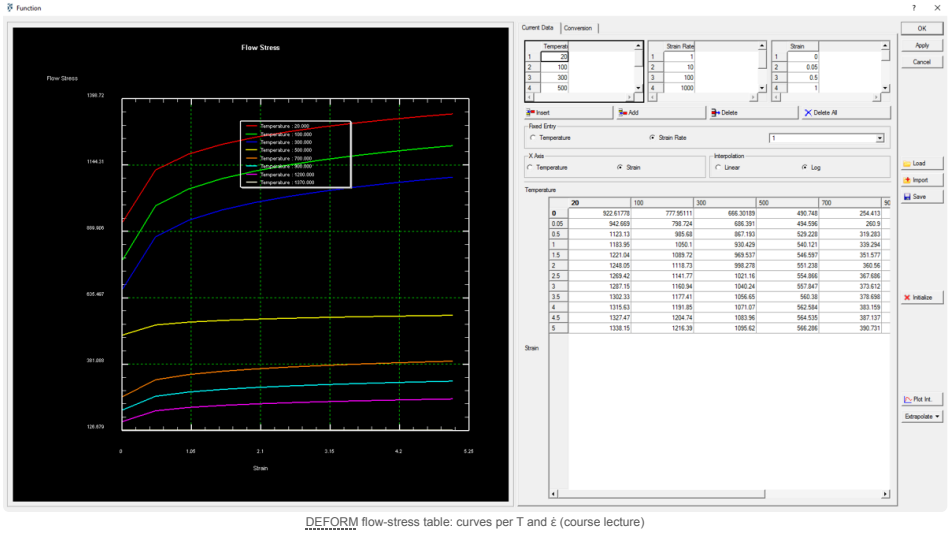


$\dot{\varepsilon} \approx 10^2 - 10^4$ s^{-1} · stress from transmitted wave, strain rate from reflected wave

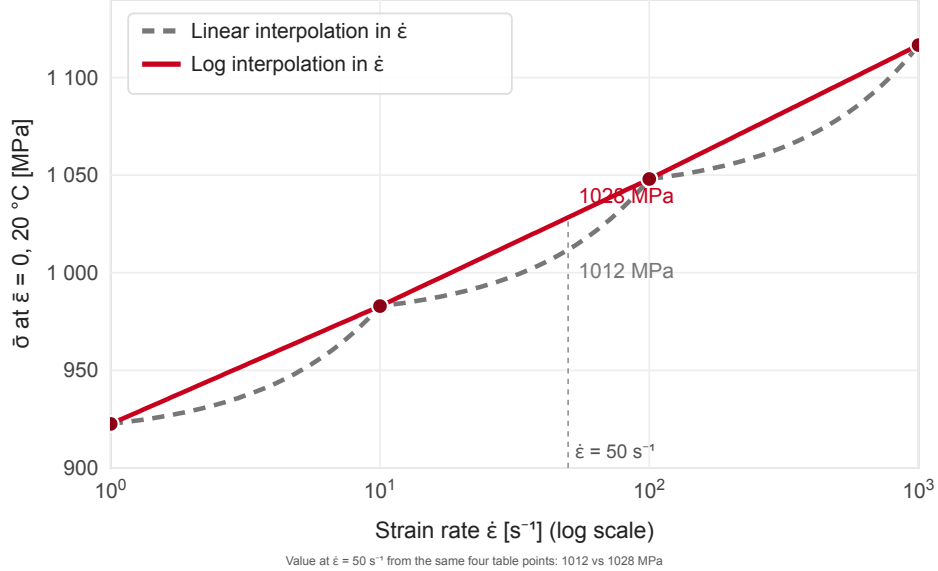
Split Hopkinson bar (SHPB)
$\dot{\varepsilon} \approx 10^2$–$10^4$ s^{-1}

Test	Strength	Limitation	DEFORM use
Tension	Standard, simple stress state	Necking limits strain	Sheet, cold forming start of curve
Compression / plane strain	Large strains, hot	Friction, barrelling → inverse correction	Bulk forming, forging
Torsion	Very large strains (ε > 5)	Gradient through radius	Hot rolling, DRX
SHPB	High rates	Small strain, adiabatic	Machining, impact (J–C)

Tabular flow stress $\bar{\sigma}(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$ and interpolation



DEFORM flow-stress table: curves per T and $\dot{\varepsilon}$ (course lecture)



Value at $\dot{\varepsilon}$ = 50 s^{-1} from the same four table points: 1012 vs 1028 MPa

- Table = curves for several **T × $\dot{\varepsilon}$**, each sampled in $\bar{\varepsilon}$ — the most common format in DEFORM libraries
- Between points DEFORM interpolates; strain rate spans decades → **logarithmic interpolation** in $\dot{\varepsilon}$ is the physically sensible choice
- Typical table: 5–8 temperatures, 4–6 strain rates (10^{-2} … 10^{2} s^{-1}), strains 0 … 2

Flow stress Extrapolation

Outside the table: extrapolation is a guess

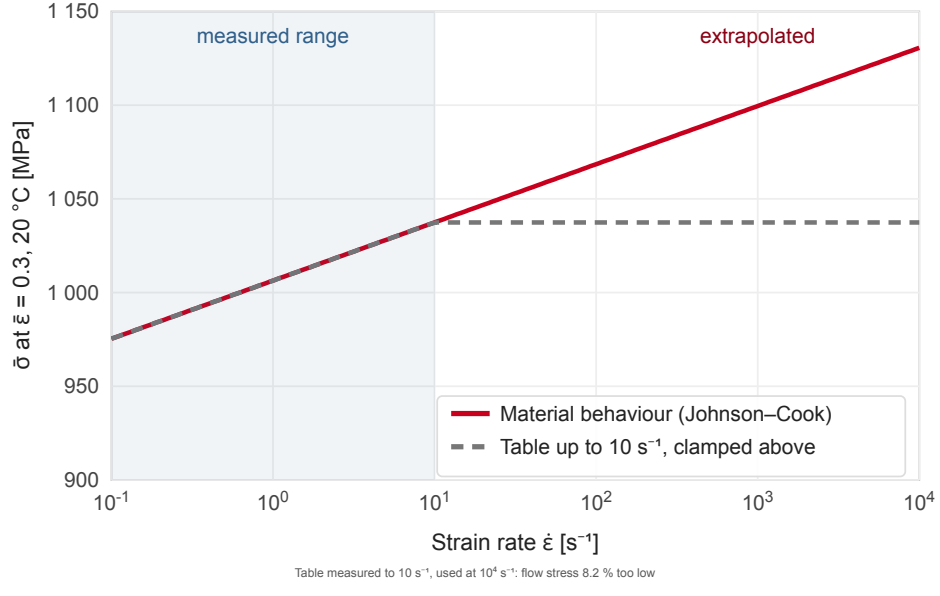
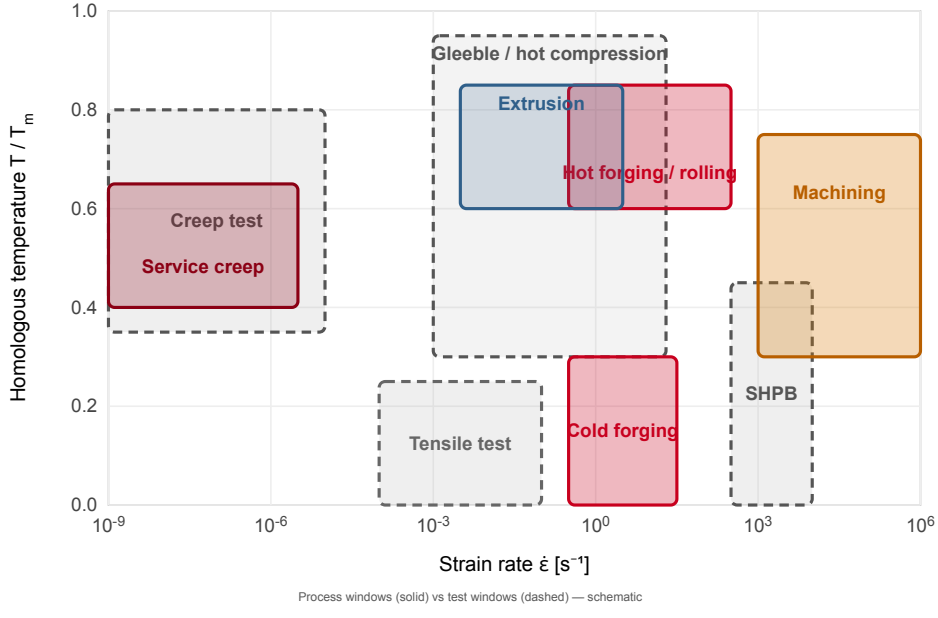


Table measured to 10 s^{-1}, used at 10^4 s^{-1}: flow stress 8.2 % too low



Process windows (solid) vs test windows (dashed) — schematic

Before running: read the post-processor ranges of ε̄, ε̇ and T and **compare them with the table limits**. DEFORM does not warn when it leaves the table.

Power law — the cold forming workhorse

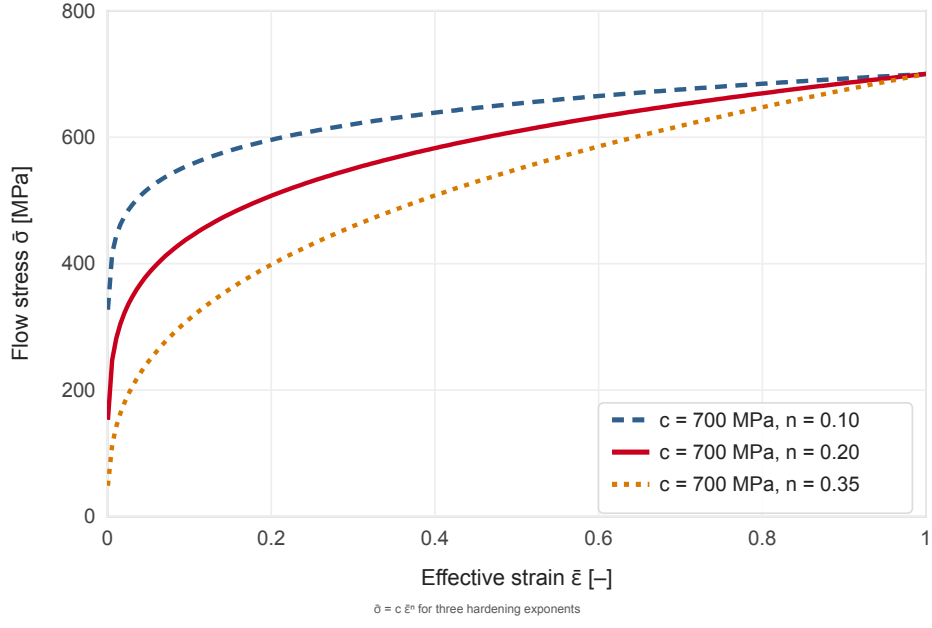
DEFORM:

$$\bar{\sigma} = c\,\bar{\varepsilon}^{\,n}\,\dot{\bar{\varepsilon}}^{\,m} + y$$

Hollomon: $\bar{\sigma} = K\bar{\varepsilon}^{\,n}$ Swift: $\bar{\sigma} = K(\varepsilon_0 + \bar{\varepsilon})^n$

- c, n, m, y may be *f(T)* → simple warm forming
- n = strain-hardening exponent = uniform strain in tension
- Pure Hollomon gives σ = 0 at ε = 0 → use y or Swift ε_0

Material (annealed)	K [MPa]	n
Low-C steel DC04	≈ 530	0.22 – 0.26
C45 (normalised)	≈ 900	0.15 – 0.18
Austenitic stainless 304	≈ 1300	0.40 – 0.50
Al 5xxx	≈ 400	0.25 – 0.30



σ̄ = c ε̄ⁿ for three hardening exponents

▼ Deep dive: 8 slides on hardening laws and the power law — press ↓

Power law ↓ 1 · The family of hardening laws

From Ludwik to Voce: one flow curve, several equations

Law	Equation	Behaviour at large ε	Typical use
Ludwik (1909)	$\sigma = \sigma_0 + K\varepsilon^n$	unbounded	general, with initial yield
Hollomon (1945)	$\sigma = K\varepsilon^n$	unbounded, σ(0) = 0	sheet n-value, quick estimates
Swift (1952)	$\sigma = K(\varepsilon_0 + \varepsilon)^n$	unbounded, σ(0) = $K\varepsilon_0^n$	sheet forming, prestrained material
Voce (1948)	$\sigma = \sigma_s - (\sigma_s - \sigma_0)\, e^{-\varepsilon/\varepsilon_c}$	saturates at σ_s	Al alloys, large strains
DEFORM power law	$\bar{\sigma} = c\,\bar{\varepsilon}^{\,n}\,\dot{\bar{\varepsilon}}^{\,m} + y$	unbounded	cold forging, heading, extrusion

- All are **empirical**: they describe the shape of the curve, not the dislocation physics behind it
- DEFORM's form = Ludwik + rate term: y plays the role of σ_0; c, n, m, y can be *f(T)*
- Mixed forms (Swift–Voce: $\sigma = w\,\sigma_{Swift} + (1-w)\,\sigma_{Voce}$) are used for AHSS sheet

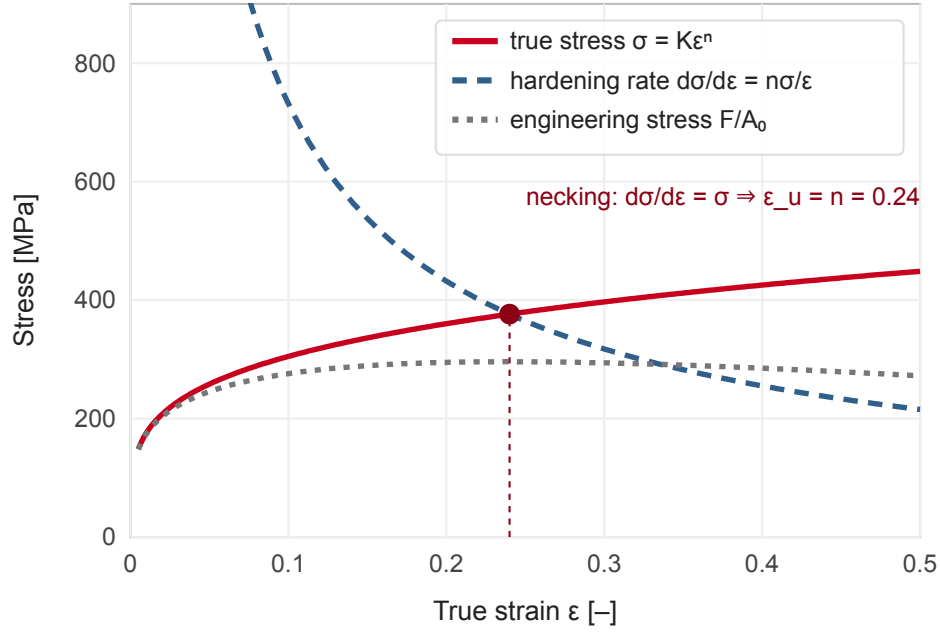
Strain-hardening exponent n and necking

Hardening rate: $\frac{d\sigma}{d\varepsilon} = nK\varepsilon^{n-1} = n\frac{\sigma}{\varepsilon}$

Considère (max. force, F = σA): $dF = 0 \Rightarrow \frac{d\sigma}{d\varepsilon} = \sigma$

Hollomon ⇒ $\varepsilon_u = n, \quad R_m = K\, n^n e^{-n}$

- n measures how long the material **spreads deformation** before it localises
- High n (austenitic stainless ≈ 0.45) → large uniform elongation, strong work hardening in drawing
- Low n (cold-worked, HSLA ≈ 0.1) → early necking, little extra strength from forming
- Check of any fit: **n ≈ uniform true strain** of the tensile test



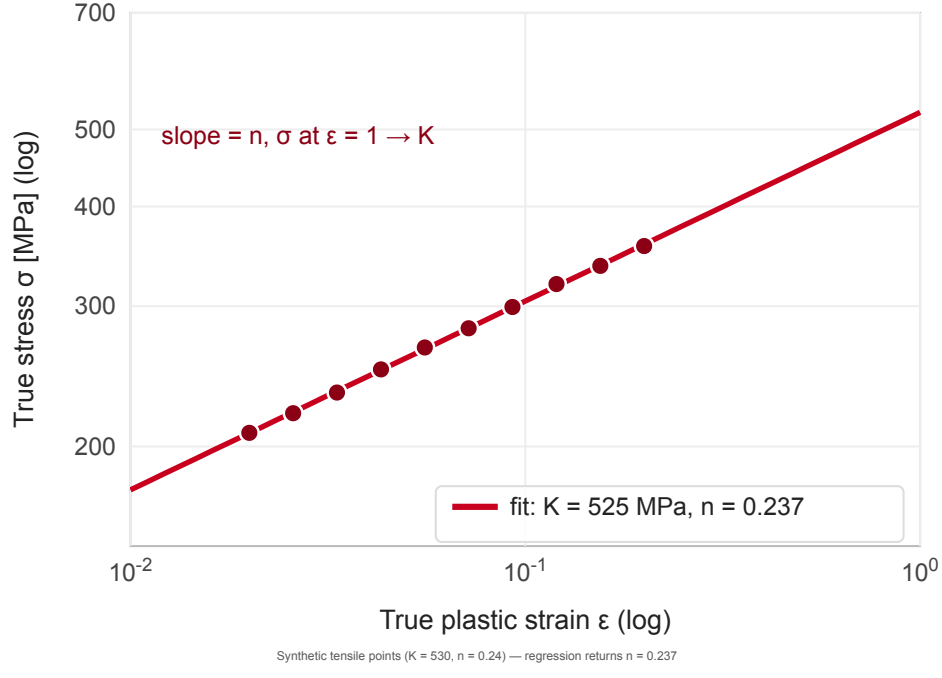
Considère construction for K = 530 MPa, n = 0.24 (DC04-type steel)

Determining K and n: the log–log plot

$$\ln \sigma = \ln K + n \ln \varepsilon$$

1. Tensile test → convert to **true** stress and **true plastic** strain (subtract σ/E)
2. Use the uniform range only, e.g. ε = 2 % … A_g (ISO 10275: n-value)
3. Linear regression of ln σ on ln ε → slope n, intercept ln K
4. Ludwik / DEFORM form: first fix y (σ_0 = yield), then fit ln(σ − y) vs ln ε
5. Cold forging: add **compression** data to ε ≈ 1 (friction-corrected) — tensile data stop at necking

Not a straight line on log–log? The material does not follow a power law (e.g. Lüders plateau, saturation) — choose Voce or a table.



Synthetic tensile points (K = 530, n = 0.24) — regression returns n = 0.237

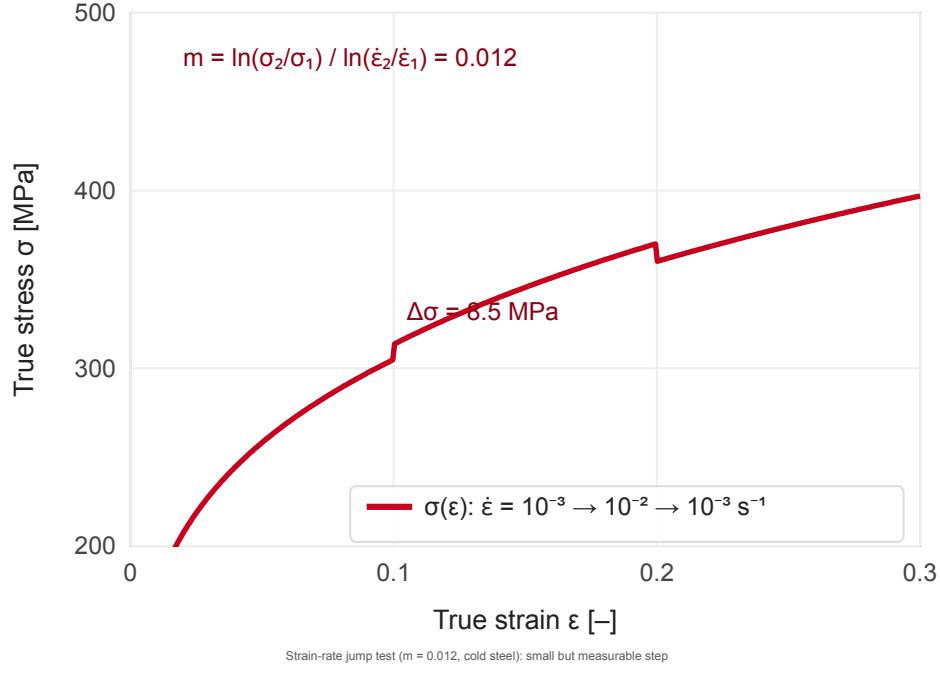
Power law ↓ 4 · Strain-rate exponent m

The strain-rate exponent m and the jump test

$$m = \left.\frac{\partial \ln \bar{\sigma}}{\partial \ln \dot{\bar{\varepsilon}}}\right|_{\varepsilon,T} \approx \frac{\ln(\sigma_2/\sigma_1)}{\ln(\dot{\varepsilon}_2/\dot{\varepsilon}_1)}$$

Condition	m	×10 in $\dot{\varepsilon}$ gives
Steel, cold (20 °C)	0.005 – 0.02	+1 … 5 %
Al alloys, cold	≈ 0 – 0.01	≈ 0 … 2 %
Warm forming (0.3–0.5 T_m)	0.03 – 0.1	+7 … 26 %
Hot forming	0.1 – 0.2	+26 … 58 %
Superplastic	≥ 0.3 – 0.5	×2 … ×3

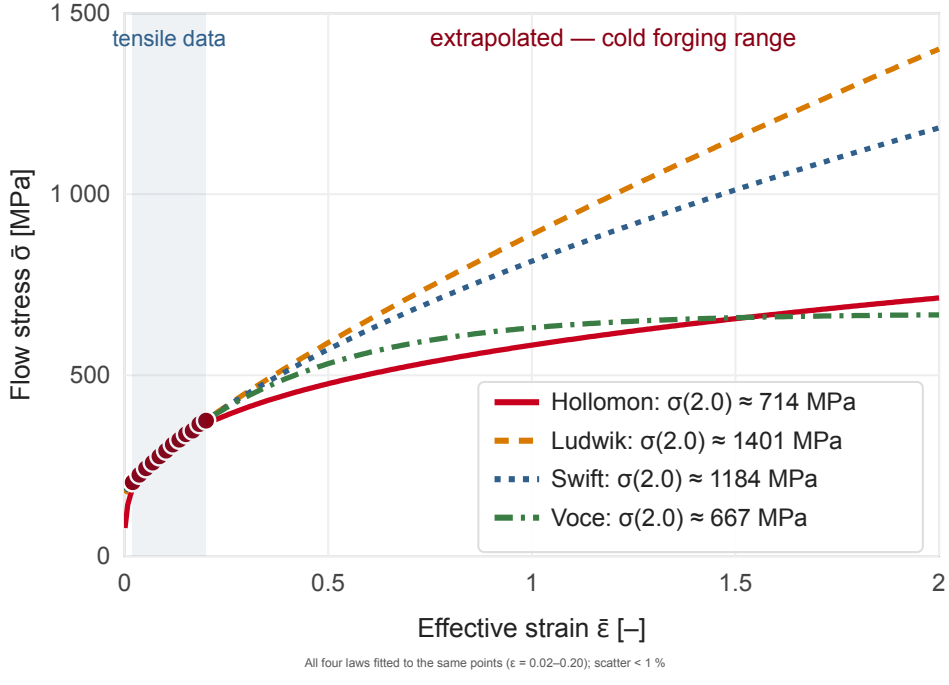
- Jump test: change $\dot{\varepsilon}$ by ×10 at constant ε and T → one specimen gives m
- Positive m **stabilises** flow (neck strain rate ↑ → stress ↑) — basis of superplasticity
- Steels at 150–350 °C: dynamic strain ageing → m < 0, serrated flow (“blue brittleness”)



Strain-rate jump test (m = 0.012, cold steel): small but measurable step

Power law ↓ 5 · The extrapolation trap

Same tensile data, four laws — and a factor of two at ε = 2



All four laws fitted to the same points (ε = 0.02–0.20); scatter < 1 %

- All four laws fit the tensile range with < 1 % error
- At ε̄ = 2 (cold forging, heading): **Voce 667**, **Hollomon 714**, **Swift 1184**, **Ludwik 1401 MPa**
- Press force and die stress scale with σ̄ → **≈ ± 35 %** error in the load, just from the choice of law
- Voce saturates (Al, Cu at large ε); power laws keep rising (steels with little recovery)

Rule: fit the law to data that **cover the strain range of the process** — compression or torsion to ε ≈ 1–2. Use tensile-only fits for sheet forming, not for cold forging.

Power law ↓ 6 · Temperature dependence

Power law with temperature: c(T), n(T), m(T), y(T)

- DEFORM lets each constant be a table **f(T)** → the power law covers warm forming
- With rising T: **c** ↓ (softer), **n** ↓ (recovery limits hardening), **m** ↑ (rate sensitivity grows)
- Linear interpolation between the T points — give enough points around phase changes
- Adiabatic heating in fast cold forging (ΔT 100–300 K) → even a “cold” simulation should be **coupled thermal** with c(T)

$$\Delta T \approx \frac{\beta}{\rho\, c_p} \int \bar{\sigma}\, d\bar{\varepsilon}, \quad \beta \approx 0.9$$

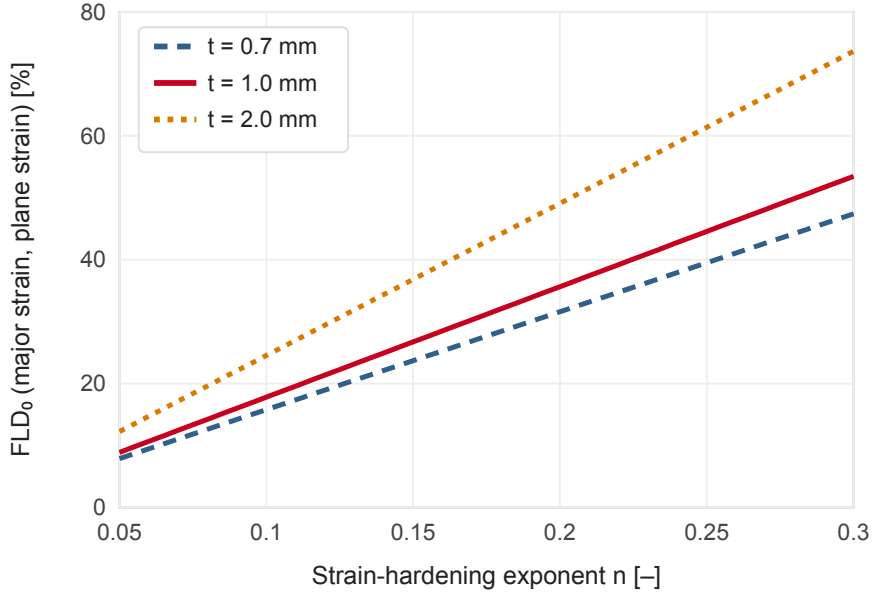
T	c [MPa]	n	m	regime
20 °C	900	0.17	0.01	cold
200 °C	850	0.15	≈ 0 (DSA)	avoid
500 °C	620	0.12	0.04	warm
700 °C	330	0.08	0.10	warm
900 °C	180	0.05	0.15	hot → use sinh / table

Illustrative trend for a medium-carbon steel — use measured values for real work

Where the power law is the right choice

Process	Why the power law
Cold forging, heading, cold extrusion	Hardening dominates, m ≈ 0; needs data to ε ≈ 2
Wire and bar drawing	Accumulated strain over passes; n sets the drawing limit
Sheet stamping, deep drawing	Swift / Hollomon with n and r-value; n → forming limit
Bending, roll forming	Moderate strains; springback needs E and hardening model too
Warm forging	c(T), n(T), m(T) tables

Keeler–Brazier (low-C steel): $FLD_0\,[\%] = (23.3 + 14.13\,t)\,\dfrac{n}{0.21}$, t in mm



Higher n → higher forming limit in plane strain (empirical, low-carbon steels)

Limits, summary and questions

Limits

- Empirical — no physics, so **no safe extrapolation** in ε, $\dot{\varepsilon}$ or T
- Unbounded hardening; Hollomon: σ(0) = 0 → use y (DEFORM) or Swift ε_0
- Rate and T only through m and f(T) tables — not for hot forming or very high rates
- No Bauschinger effect, no anisotropy — those come from the yield function and hardening rule (block 5)

Summary

1. Ludwik, Hollomon, Swift, Voce: same data, different large-strain behaviour
2. n = uniform strain (Considère); m from jump tests
3. K, n from a log–log fit in the uniform range
4. Fit to data covering the process strain — or the load can be off by ≈ ± 35 %

Check yourself

1. Derive ε_u = n from the Considère criterion.
2. Why does Hollomon need a y-term or Swift's ε_0 in FEM?
3. m = 0.15: how much does σ change if $\dot{\varepsilon}$ rises 100×?
4. Your fit used tensile data to ε = 0.2. Can you use it for cold heading to ε = 2.5?
5. Which law saturates, and for which materials is that realistic?

Press ↑ to return to the main lecture, or → to continue with Johnson–Cook.

Flow stress Johnson–Cook

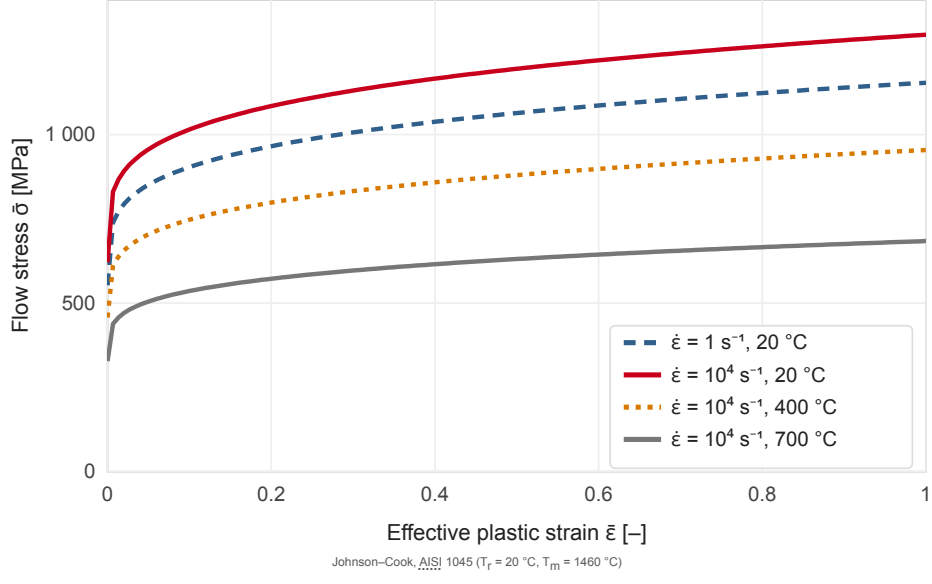
Johnson–Cook — high rates, machining, impact

$$\bar{\sigma} = \underbrace{(A + B\bar{\varepsilon}^{\,n})}_{\text{hardening}}\underbrace{(1 + C\ln\dot{\varepsilon}^{*})}_{\text{rate}}\underbrace{(1 - T^{*m})}_{\text{softening}}$$

$$\dot{\varepsilon}^{*} = \dot{\bar{\varepsilon}}/\dot{\varepsilon}_0, \quad T^{*} = \frac{T - T_r}{T_m - T_r}$$

AISI 1045	A	B	n	C	m
Jaspers & Dautzenberg	553 MPa	601 MPa	0.234	0.0134	1.0

- Three **independent** factors — easy to fit, no coupling of T and $\dot{\varepsilon}$
- DEFORM form adds $\dot{\varepsilon}^{*\alpha}$ and $(D - E\,T^{*m})$ for more freedom
- Fails near T_m and for DRX-dominated hot forming



Johnson–Cook, AISI 1045 (T_r = 20 °C, T_m = 1460 °C)

▼ Deep dive: 9 slides on Johnson–Cook — press ↓

Johnson–Cook ↓ 1 · Origin

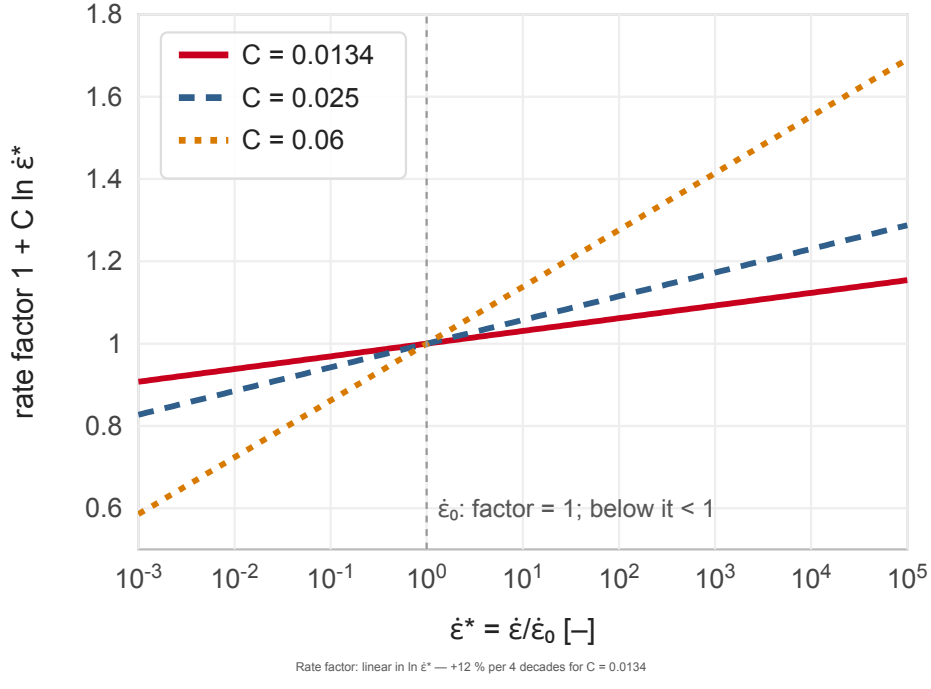
A model built for impact codes

- **G. R. Johnson & W. H. Cook** (Honeywell): “A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures”, *7th Int. Symp. on Ballistics*, The Hague, **1983**
- Goal: a model **simple enough for explicit impact codes** (EPIC) with constants obtainable from a few standard tests
- Tests: **torsion** (quasi-static to ≈ 400 s^{-1}, large shear strains), static tension, **Hopkinson-bar tension** at elevated T
- Validation: **Taylor cylinder impact** (strains > 2, rates > 10^5 s^{-1}) — not used for fitting
- Companion **fracture model**: Johnson & Cook, *Eng. Fract. Mech.* 21 (1985) 31

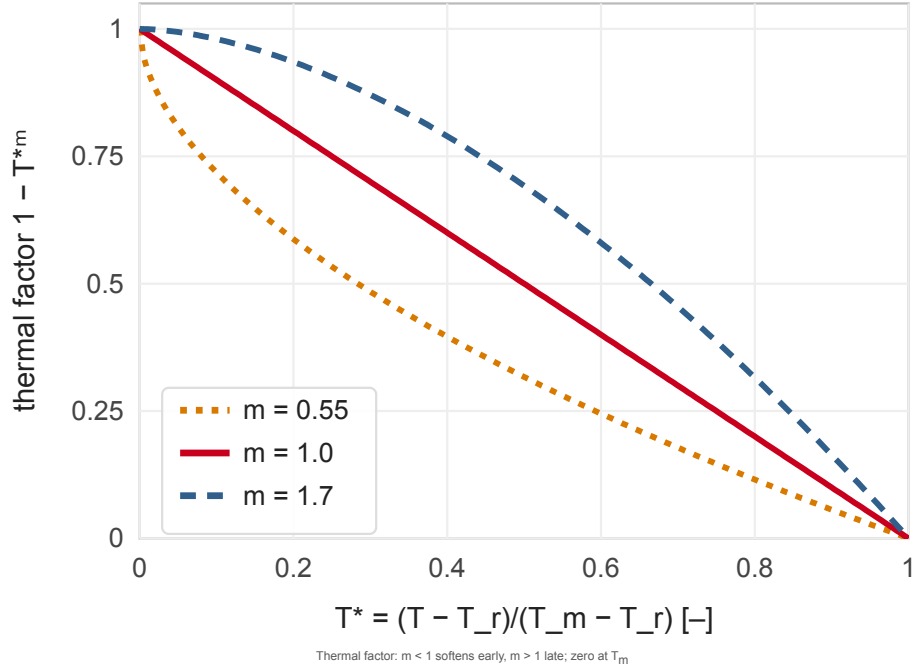
$$\bar{\sigma} = \left[A + B\bar{\varepsilon}^{\,n}\right]\left[1 + C\ln\dot{\varepsilon}^{*}\right]\left[1 - T^{*m}\right]$$

Symbol	Meaning	From test
A	yield stress at $\dot{\varepsilon}_0$, T_r	quasi-static
B, n	strain hardening	quasi-static
C	strain-rate sensitivity	SHPB / torsion
m	thermal softening	tests at T
$\dot{\varepsilon}_0$, T_r, T_m	reference rate (often 1 s^{-1}), reference and melting temperature	chosen / handbook

Three independent multipliers



Rate factor: linear in ln ε̇* — +12 % per 4 decades for C = 0.0134



Thermal factor: m < 1 softens early, m > 1 late; zero at T_m

- **Hardening** [A + Bεⁿ]: Ludwik law — the flow curve at $\dot{\varepsilon}_0$ and T_r
- **Rate** [1 + C ln ε̇*]: below $\dot{\varepsilon}_0$ the factor is < 1 (choose $\dot{\varepsilon}_0$ = lowest test rate, or clip ε̇* ≥ 1); constant C → no upturn above ≈ 10^3–10^4 s^{-1}
- **Thermal** [1 − T*ᵐ]: T* < 0 below T_r gives an undefined power for non-integer m → codes set T* = 0

Johnson–Cook ↓ 3 · Published constants

Constants from the original paper and later work

Material	A [MPa]	B [MPa]	n	C	m	T_m [K]	Source
OFHC copper	90	292	0.31	0.025	1.09	1356	J–C 1983
Cartridge brass	112	505	0.42	0.009	1.68	1189	J–C 1983
Nickel 200	163	648	0.33	0.006	1.44	1726	J–C 1983
1006 steel	350	275	0.36	0.022	1.00	1811	J–C 1983
4340 steel	792	510	0.26	0.014	1.03	1793	J–C 1983
2024-T351 Al	265	426	0.34	0.015	1.00	775	J–C 1983
AISI 1045	553	601	0.234	0.0134	1.00	1733	Jaspers & Dautzenberg 2002

- Constants belong to **one material condition** (heat treatment, hardness) and one fitting range
- Large scatter between published sets for the same grade — always check σ(ε) at your $\dot{\varepsilon}$ and T before use
- C is small (0.006–0.06): rate effects are moderate; m ≈ 1 means roughly linear softening with T

Johnson–Cook ↓ 4 · Calibration

Determining A, B, n, C and m

1 · A: yield stress at $\dot{\varepsilon}_0$ and T_r (quasi-static tension/compression)

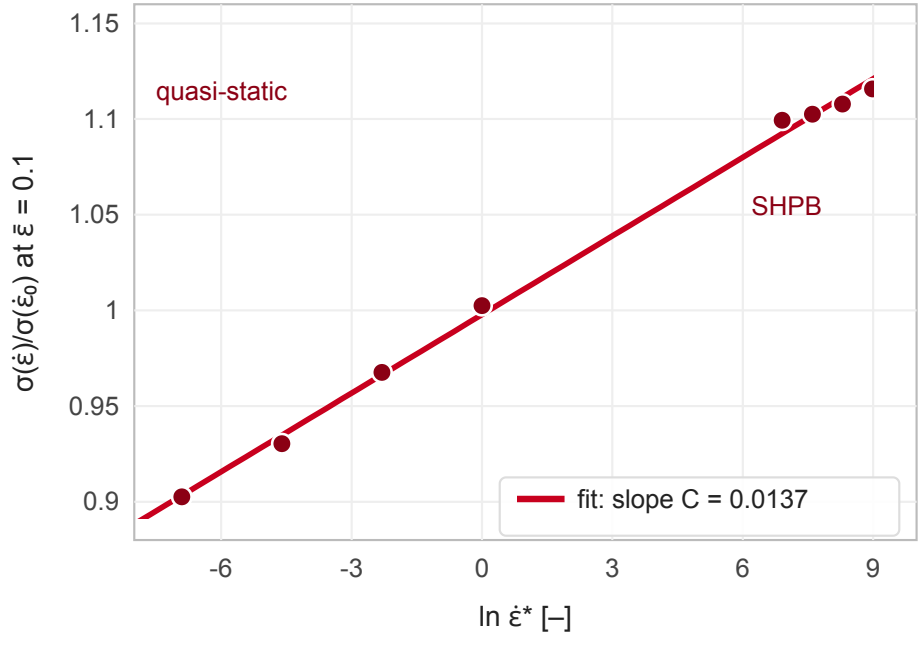
2 · B, n: $\ln(\sigma - A) = \ln B + n \ln \varepsilon$ — log–log fit of the same test

3 · C: at fixed ε, $\sigma/\sigma_{ref} = 1 + C \ln \dot{\varepsilon}^*$ → **slope (SHPB at 10^3–10^4 s^{-1})**

4 · m: at fixed ε, $\dot{\varepsilon}$: $\ln(1 - \sigma/\sigma_{ref}) = m \ln T^*$

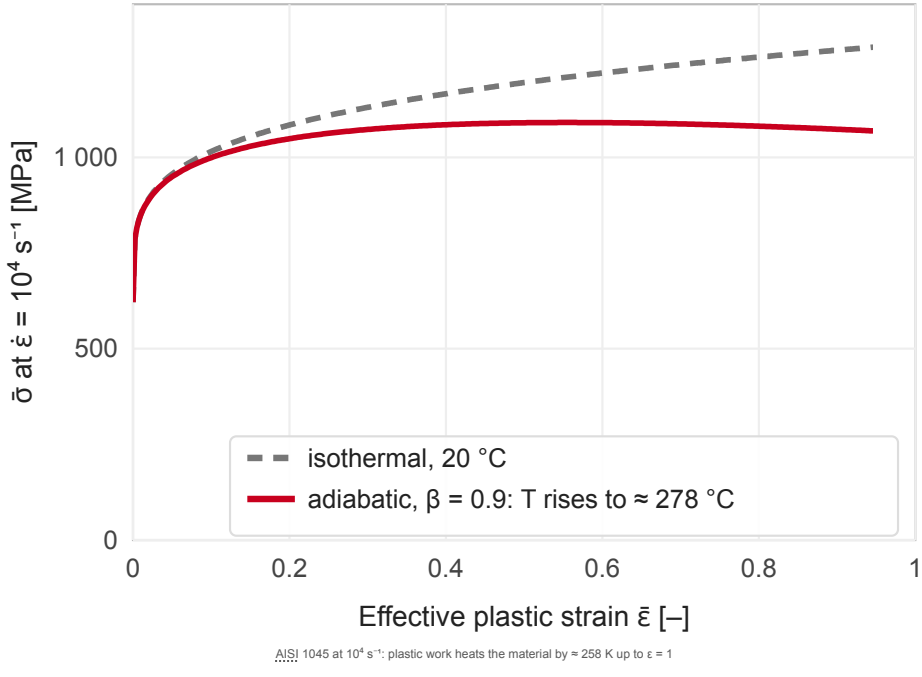
5 · Check: correct SHPB curves for **adiabatic heating**; validate on an independent test (Taylor, cutting force)

Alternative: fit all five constants together by least squares (or inverse FEM of the tests) — better overall error, less transparent



Step 3 with synthetic points (C = 0.0134): regression gives C = 0.0137

Thermal softening during fast deformation



AISI 1045 at 10^4 s^{-1}: plastic work heats the material by ≈ 258 K up to ε = 1

$$\rho\, c_p\, \dot{T} = \beta\, \bar{\sigma}\, \dot{\bar{\varepsilon}}, \qquad \beta \approx 0.9 \text{ (Taylor–Quinney)}$$

- Above ≈ 10–100 s^{-1} there is no time for heat conduction → **adiabatic**
- J–C's thermal bracket then **lowers** the hardening curve as T rises
- Hardening and softening compete → flat or falling curve → **adiabatic shear bands**, chip segmentation
- In FEM: always a **coupled thermo-mechanical** run with ρ, c_p(T), k(T); never fit J–C to SHPB curves as if they were isothermal

Limitations — and the modified Johnson–Cook models

Known limitations

- **No coupling**: rate sensitivity does not depend on T, hardening does not depend on $\dot{\varepsilon}$ or T
- Constant C → misses the **upturn** above 10^3–10^4 s^{-1} (dislocation drag)
- Accuracy falls at high T; no recovery, recrystallisation or phase change
- Unbounded hardening (Ludwik) at large strains
- BCC metals (yield strongly T-dependent) → Zerilli–Armstrong often better

Modified forms

DEFORM: $\bar{\sigma} = (A + B\bar{\varepsilon}^{\,n})(1 + C \ln \dot{\varepsilon}^{*})\, \dot{\varepsilon}^{*\alpha}\, (D - E\, T^{*m})$

Lin et al. 2010: $\bar{\sigma} = (A_1 + B_1\varepsilon + B_2\varepsilon^2)(1 + C_1 \ln \dot{\varepsilon}^{*})\, e^{(\lambda_1 + \lambda_2 \ln \dot{\varepsilon}^{*})(T - T_r)}$

Quadratic rate term: $1 + C_1 \ln \dot{\varepsilon}^{*} + C_2 (\ln \dot{\varepsilon}^{*})^2$

- Each adds coupling or curvature — and constants; the original stays the default in most codes

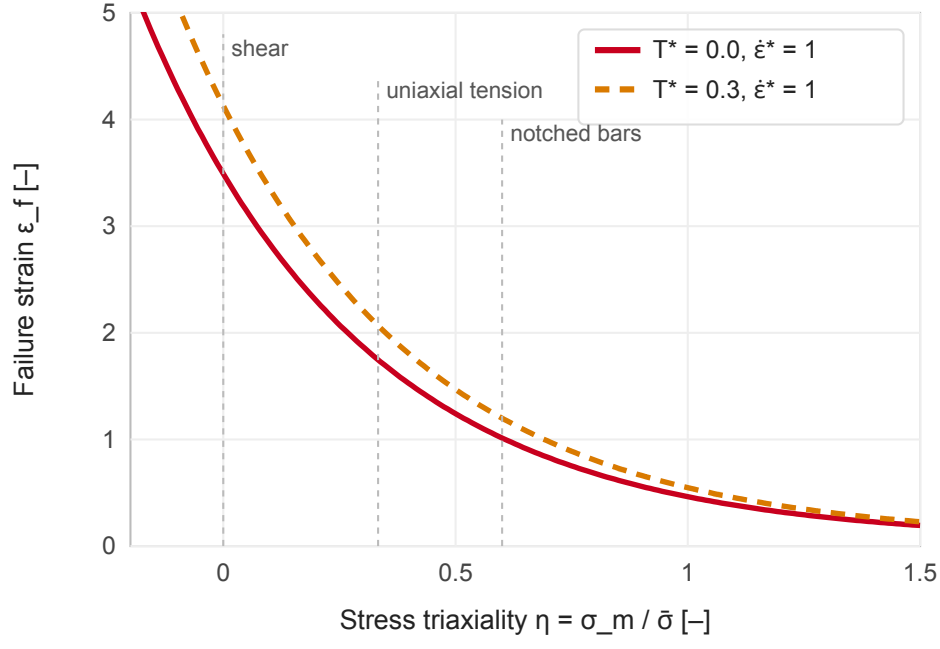
Johnson–Cook ↓ 7 · Fracture model

The Johnson–Cook fracture (damage) model

$$\varepsilon_f = \left[D_1 + D_2\, e^{D_3 \eta}\right]\left[1 + D_4 \ln \dot{\varepsilon}^*\right]\left[1 + D_5 T^*\right]$$

$$D = \sum \frac{\Delta \bar{\varepsilon}}{\varepsilon_f} \geq 1 \;\Rightarrow\; \text{fracture (element deletion)}$$

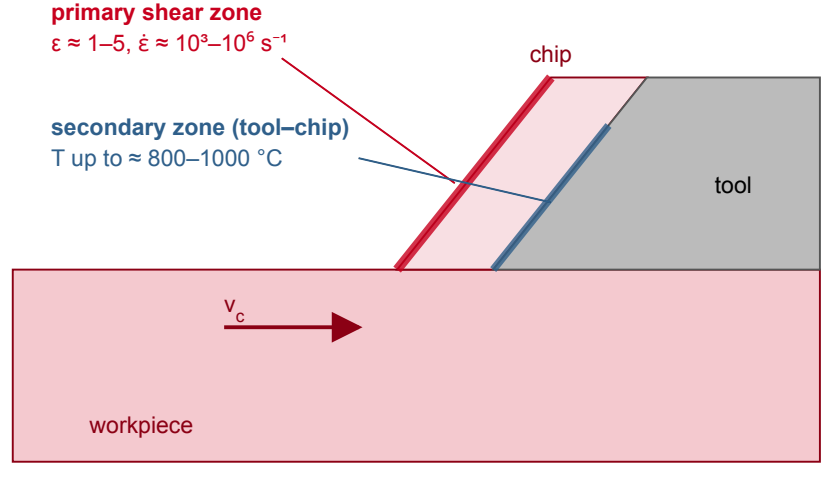
- $\eta = \sigma_m / \bar{\sigma}$ stress triaxiality: shear 0, tension ⅓, notched bars 0.5–1
- Same structure as the flow model: triaxiality × rate × temperature
- 4340 steel (J–C 1985): D_1 = 0.05, D_2 = 3.44, D_3 = −2.12, D_4 = 0.002, D_5 = 0.61
- Calibration: notched round bars (η), torsion (η ≈ 0), SHPB, hot tests
- Used for chip separation, blanking, ballistic perforation; compare with Cockcroft–Latham (main slide 41)



Failure strain falls steeply with triaxiality (4340 steel, J–C 1985)

Where Johnson–Cook is used

Operation	Why J–C	Typical $\dot{\varepsilon}$, T
Machining (turning, milling, drilling) — DEFORM-3D Machining	Chip formation, cutting forces, tool temperature; J–C + J–C damage for segmentation	10^3–10^6 s^{-1}, up to ≈ 1000 °C
Impact, ballistics, Taylor test	Original purpose; robust in explicit codes	10^3–10^5 s^{-1}, adiabatic
Crash of car structures	Rate hardening of steel and Al sheet	1–10^3 s^{-1}
High-speed forming (electromagnetic, explosive, hammer)	Rate + adiabatic heating	10^2–10^4 s^{-1}
Blanking, shearing, punching	Localised shear, damage model	10–10^3 s^{-1}
Hot forging	**Not recommended** — use sinh law or tables	> 0.6 T_m



orthogonal cutting (schematic)

Orthogonal cutting: extreme strain rate in the primary shear zone, high temperature at the tool–chip interface

Practical tips, summary and questions

Tips for DEFORM

- Choose the J–C entry in Plastic → Flow stress; set D = 1, E = 1, α = 0 to get the original form
- Use T_r, T_m in the problem's temperature unit; set $\dot{\varepsilon}_0$ to the reference rate of the published set
- Run **coupled thermal**; enter k(T), $\rho c_p(T)$
- Plot σ(ε) at the process $\dot{\varepsilon}$ and T in the material window before running

Summary

1. Three independent brackets: hardening × rate × softening
2. Five constants from quasi-static, SHPB and hot tests
3. Strong with adiabatic heating: shear bands, chips, impact
4. Weak where T and $\dot{\varepsilon}$ interact: hot forming, very high rates

Check yourself

1. Which test gives C, and which gives m?
2. What does the rate bracket do below $\dot{\varepsilon}_0$, and how do you avoid it?
3. Why must SHPB curves be corrected before fitting?
4. Name two physical effects J–C cannot describe.
5. How does triaxiality change the J–C failure strain?

Press ↑ to return to the main lecture, or → to continue with Zerilli–Armstrong.

Flow stress Zerilli–Armstrong

Zerilli–Armstrong — physics of dislocations, BCC vs FCC

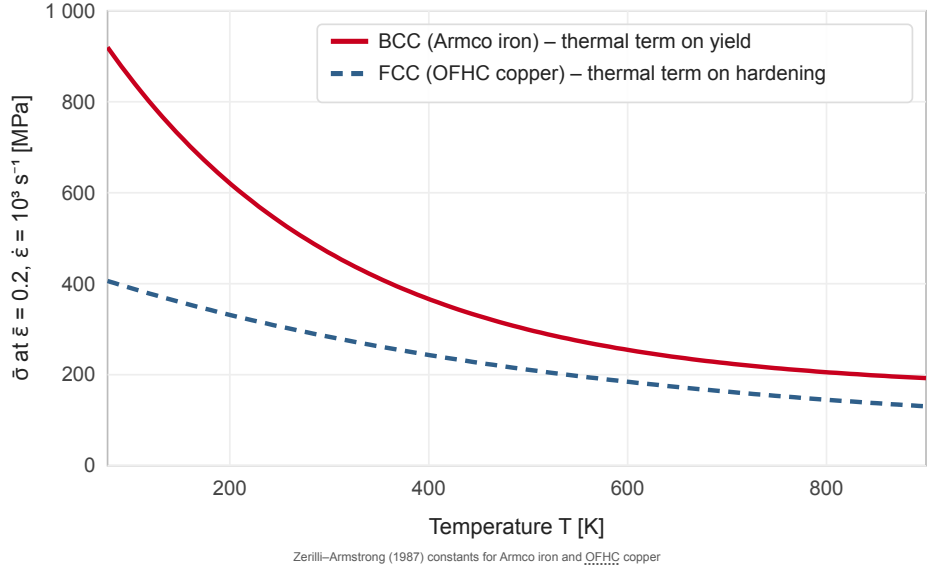
BCC:

$$\bar{\sigma} = C_0 + C_1 e^{-C_3 T + C_4 T \ln \dot{\varepsilon}} + C_5\, \bar{\varepsilon}^{\,n}$$

FCC:

$$\bar{\sigma} = C_0 + C_2\, \bar{\varepsilon}^{1/2}\, e^{-C_3 T + C_4 T \ln \dot{\varepsilon}}$$

- Based on thermally activated dislocation motion; T in kelvin
- **BCC** (steels, Ta, Mo): T and $\dot{\varepsilon}$ affect the *yield stress* (Peierls barrier)
- **FCC** (Cu, Al, Ni, austenite): T and $\dot{\varepsilon}$ affect the *hardening*
- DEFORM list: $\bar{\sigma} = a + c_1 \exp(-c_3 T + c_4 T \ln \dot{\bar{\varepsilon}}) + c_5 \bar{\varepsilon}^{\,n}$



Zerilli–Armstrong (1987) constants for Armco iron and OFHC copper

▼ Deep dive: 10 slides on Zerilli–Armstrong — press ↓

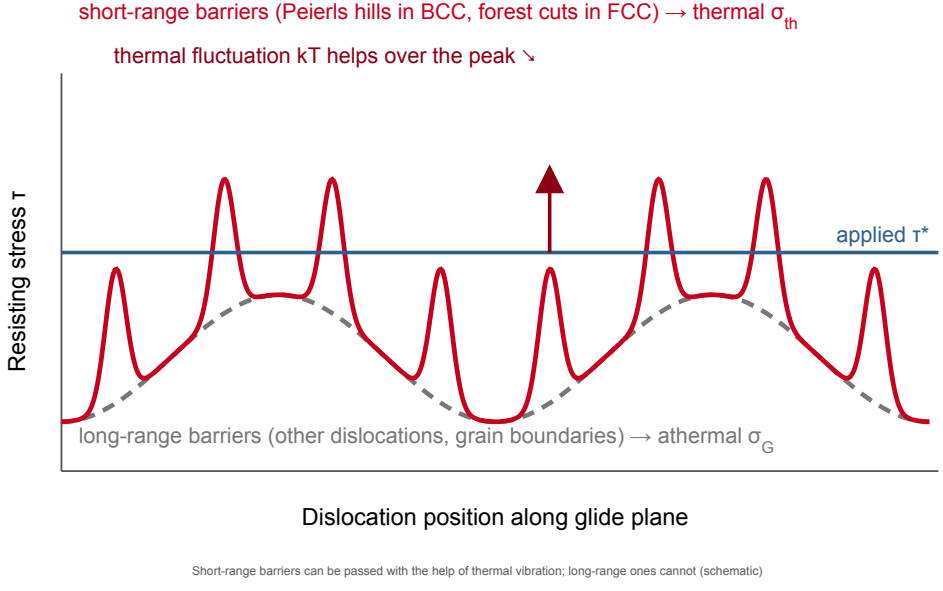
Zerilli–Armstrong ↓ 1 · Origin and motivation

Why a dislocation-based model?

- **F. J. Zerilli & R. W. Armstrong**, Naval Surface Warfare Center, *J. Appl. Phys.* 61 (1987) 1816
- Goal: flow stress for **material dynamics** codes — impact, penetration, explosive loading
- Empirical models (Johnson–Cook 1983) treat T and $\dot{\varepsilon}$ as **independent** factors and use the **same form for every metal**
- Experiments showed: in **iron** the yield stress depends strongly on T and $\dot{\varepsilon}$; in **copper** mainly the hardening does
- Idea: derive the form from **thermally activated dislocation motion** — and let the **crystal lattice** decide the form

	Johnson–Cook	Zerilli–Armstrong
Basis	Empirical fit	Dislocation mechanics
T–$\dot{\varepsilon}$ coupling	None (product of brackets)	Built in: $e^{-C_3 T + C_4 T \ln \dot{\varepsilon}}$
Lattice	Same form for all	BCC / FCC / HCP forms
Grain size	Not explicit	Hall–Petch term $k\, d^{-1/2}$
Temperature	Homologous T*, needs T_m	Absolute T [K]
Constants	5 (A, B, n, C, m)	5–6 ($C_0 \ldots C_5$, n)

Thermal activation: dislocations jump over barriers



Short-range barriers can be passed with the help of thermal vibration; long-range ones cannot (schematic)

$$\bar{\sigma} = \underbrace{\sigma_G}_{\text{athermal}} + \underbrace{\sigma_{th}(T,\dot{\varepsilon})}_{\text{thermal}} + \underbrace{k\,d^{-1/2}}_{\text{Hall–Petch}}$$

Arrhenius rate of jumps:

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{\Delta G}{k_B T}\right)$$

- ΔG = energy the stress must *not* supply; the rest comes from thermal vibration
- Higher T → more help from kT → **lower** σ_{th}
- Higher $\dot{\varepsilon}$ → less time to wait for a fluctuation → **higher** σ_{th}
- σ_G: long-range stress fields, solutes, initial dislocation density

From the Arrhenius law to the ZA equation

① **Thermal stress from thermal activation (activation area A)**

$$\sigma_{th} = \frac{M\,\Delta G_0}{A\,b}\, e^{-\beta T}, \qquad \beta = \beta_0 - \beta_1 \ln \dot{\varepsilon}$$

② **BCC: A independent of strain (Peierls barrier)**

$$\bar{\sigma} = C_0 + C_1\, e^{-C_3 T + C_4 T \ln \dot{\varepsilon}} + C_5\, \bar{\varepsilon}^{\,n}$$

③ **FCC: A ∝ $\bar{\varepsilon}^{-1/2}$ (forest density ρ ∝ $\bar{\varepsilon}$)**

$$\bar{\sigma} = C_0 + C_2\, \bar{\varepsilon}^{1/2}\, e^{-C_3 T + C_4 T \ln \dot{\varepsilon}}$$

④ **Common athermal term**

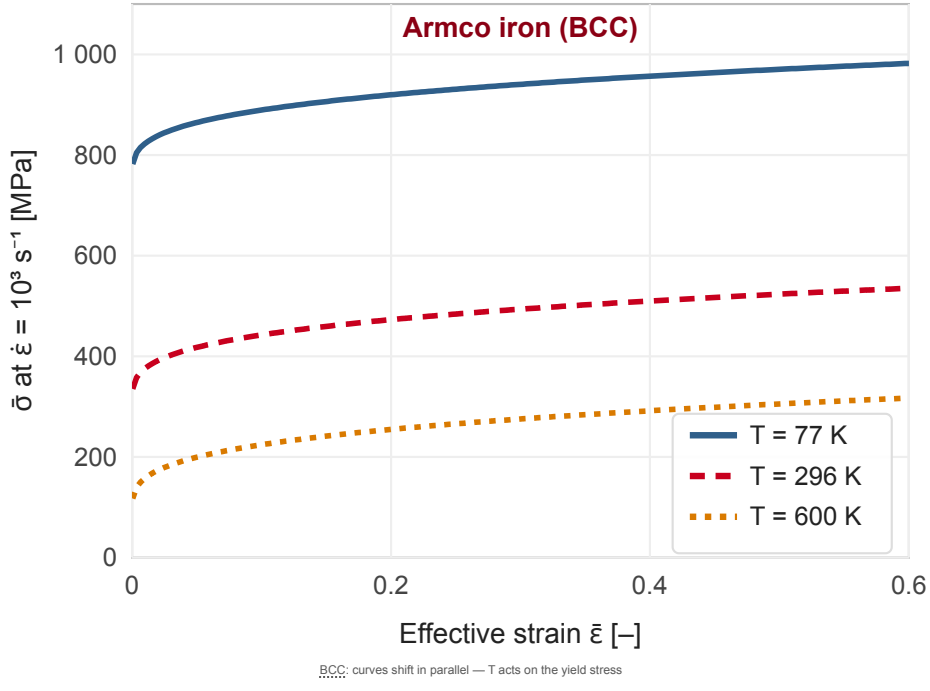
$$C_0 = \Delta\sigma'_G + k\, d^{-1/2}$$

Constant	Physical meaning	Unit
C_0	athermal stress + Hall–Petch	MPa
C_1 (BCC)	$M\Delta G_0/(Ab)$ — thermal yield amplitude	MPa
C_2 (FCC)	thermal hardening amplitude	MPa
$C_3 = \beta_0$	thermal softening	K^{-1}
$C_4 = \beta_1$	T–$\dot{\varepsilon}$ coupling	K^{-1}
C_5, n (BCC)	athermal strain hardening	MPa, –
k, d	Hall–Petch slope, grain size	MPa·$mm^{½}$, mm

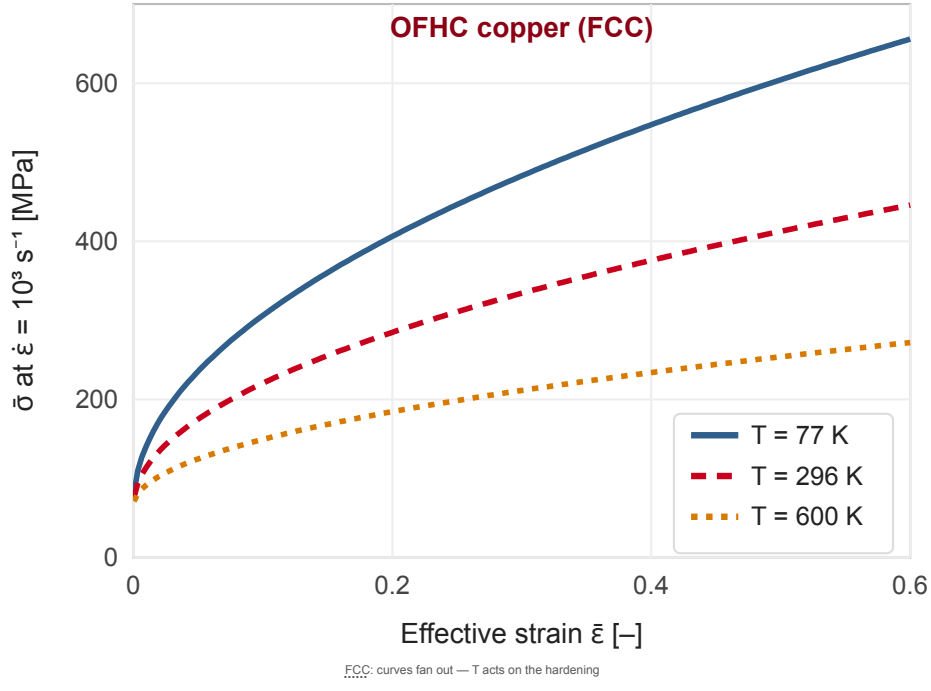
- M Taylor factor, b Burgers vector, ΔG_0 activation energy at 0 K
- $\dot{\varepsilon}$ in s^{-1} (normalised by $\dot{\varepsilon}_0 = 1\ s^{-1}$); **T always in kelvin**

Zerilli–Armstrong ↓ 4 · Crystal structure decides

BCC vs FCC: the same temperatures, different curves



BCC: curves shift in parallel — T acts on the yield stress

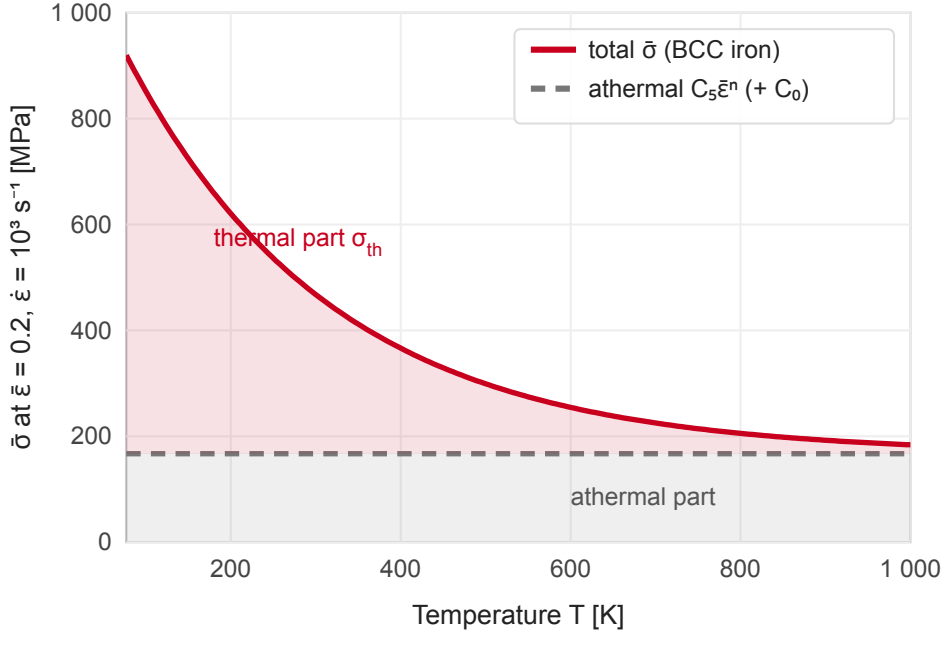


FCC: curves fan out — T acts on the hardening

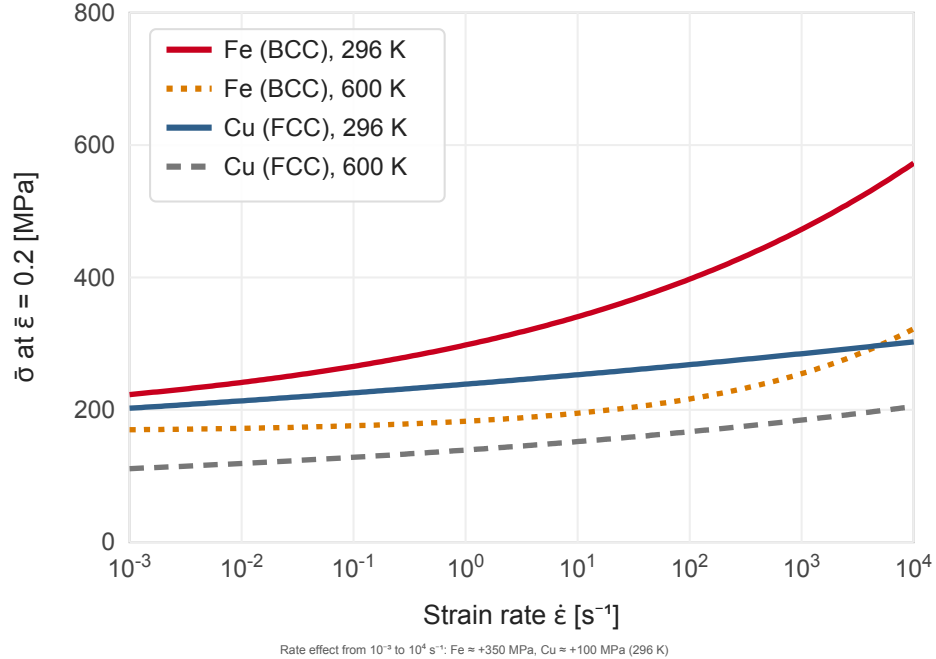
- **BCC** (ferritic steels, Fe, Mo, W, Ta, Nb): high T- and ε̇-sensitive yield, low-temperature brittleness
- **FCC** (Cu, Al, Ni, austenitic steels): yield nearly independent of T; hardening rate falls with T
- **HCP** (Ti, Mg, Zr): both effects → combined BCC + FCC terms (Zerilli & Armstrong, 1990s)

Zerilli–Armstrong ↓ 5 · Sensitivities

Where the T and ε̇ sensitivity comes from



Iron: thermal part vanishes with T; athermal part stays



Rate effect from 10^{-3} to 10^{4} s^{-1}: Fe ≈ +350 MPa, Cu ≈ +100 MPa (296 K)

Rate sensitivity: $\frac{\partial \bar{\sigma}}{\partial \ln \dot{\varepsilon}} = C_4\, T\, \sigma_{th}$ — grows with T at first, then falls as $\sigma_{th} \to 0$; T–ε̇ trade-off: $\Delta T \leftrightarrow \Delta \ln \dot{\varepsilon} \cdot \frac{C_4 T}{C_3 - C_4 \ln \dot{\varepsilon}}$

Zerilli–Armstrong ↓ 6 · Constants and variants

Published constants and model variants

Metal	C_0	C_1 / C_2	C_3 [K⁻¹]	C_4 [K⁻¹]	C_5	n
OFHC Cu (FCC)	≈ 65*	C_2 = 890	0.0028	$1.15 \cdot 10^{-4}$	–	½
Armco Fe (BCC)	k $d^{-½}$	C_1 = 1033	0.00698	$4.15 \cdot 10^{-4}$	266	0.289
Tantalum (BCC)	30	C_1 = 1125	0.00535	$3.27 \cdot 10^{-4}$	310	0.44

Stresses in MPa. *Cu: σ_G = 46.5 MPa + k $d^{-½}$ with k = 5 MPa·$mm^{½}$, d = 0.073 mm. Sources: Zerilli & Armstrong 1987; Banerjee 2005 (arXiv cond-mat/0512466); Ta set as quoted in arXiv 1105.5514

DEFORM (dropdown list)

$$\bar{\sigma} = a + c_1 \exp(-c_3 T + c_4 T \ln \dot{\bar{\varepsilon}}) + c_5\, \bar{\varepsilon}^{\,n}$$ — the BCC form

Modified ZA for hot forming (Samantaray et al. 2009)

$$\bar{\sigma} = (C_1 + C_2 \bar{\varepsilon}^{\,n}) \exp\{-(C_3 + C_4 \bar{\varepsilon}) T^* + (C_5 + C_6 T^*) \ln \dot{\varepsilon}^*\}$$

- $T^* = T - T_{ref}$, $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_{ref}$ — fitted to Gleeble hot-compression data
- Used for 9Cr-1Mo, Ti-modified austenitic SS (R ≈ 0.995, AARE ≈ 5 %), Ti alloys, Al alloys, roll steels
- Loses the dislocation meaning — it is a **semi-empirical** fit; extensions (IMZA) add more couplings

Implemented in

- DEFORM (built-in) · LS-DYNA (*MAT_MODIFIED_ZERILLI_ARMSTRONG) · Ansys Explicit / Autodyn · Abaqus via VUMAT

Zerilli–Armstrong ↓ 7 · Calibration

How to determine the constants (BCC form)

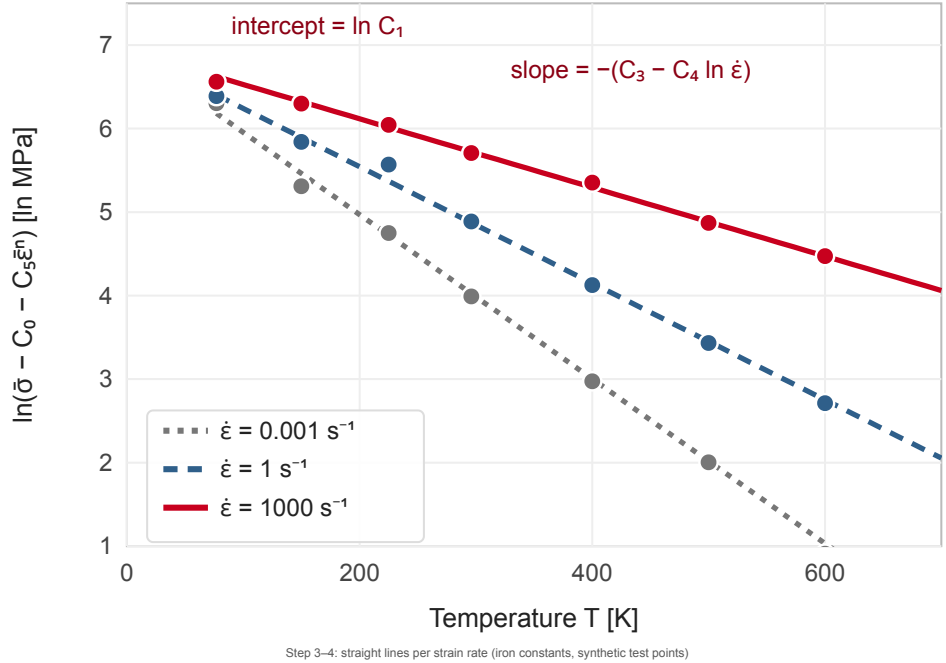
1 · Test matrix: quasi-static (10^{-3}–10^{-1} s^{-1}) + SHPB (10^{3} s^{-1}) at 3–5 temperatures, 77 K … 0.5 T_m

2 · Athermal part: C_5, n from hardening $\bar{\sigma} - \sigma_y$ vs $\bar{\varepsilon}$ (log–log); C_0 from grain size or high-T plateau

3 · Linearise: $\ln(\bar{\sigma} - C_0 - C_5\bar{\varepsilon}^n) = \ln C_1 - (C_3 - C_4 \ln \dot{\varepsilon})\, T$

4 · Slopes for each $\dot{\varepsilon}$ → plot slope vs ln $\dot{\varepsilon}$: intercept C_3, slope C_4

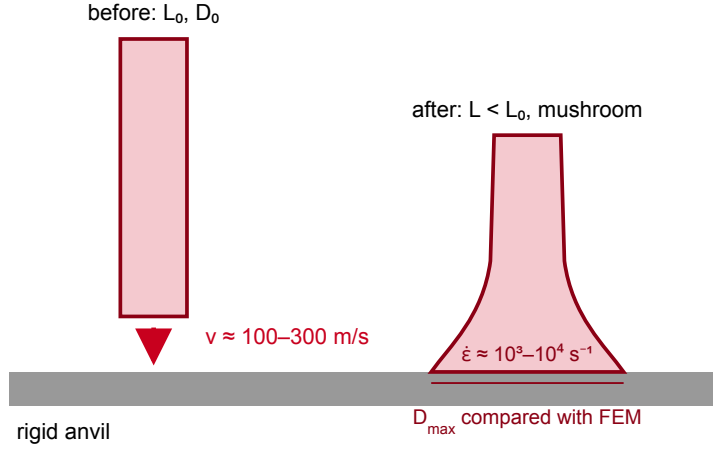
5 · Check: AARE, R; simulate a **Taylor test** or the compression test in FEM (inverse correction for adiabatic heating)



Step 3–4: straight lines per strain rate (iron constants, synthetic test points)

Where and in which operations ZA is used

Operation	Why ZA	Typical $\dot{\varepsilon}$, T
Taylor impact test (model validation)	Original validation of ZA for Cu and Fe; one test checks the whole ε–$\dot{\varepsilon}$–T field	10^3–10^4 s^{-1}, 296–1200 K
Ballistics, penetrators, EFP liners	BCC Fe, Ta, W: strong rate effect at high rates	10^4 s^{-1}, adiabatic
High-speed machining	Chip formation, shear bands in steels; compared with J–C	10^3–10^6 s^{-1}, up to 1000 °C
High-speed forming (electromagnetic, explosive, hammer)	Rate hardening of ferritic sheet and tube	10^2–10^4 s^{-1}
Cold heading, crash, low-T service	BCC yield vs FCC hardening; ductile–brittle transition	1–10^3 s^{-1}, 200–400 K
Hot forging, rolling (modified ZA)	Semi-empirical fit of Gleeble data	10^{-3}–10 s^{-1}, > 0.6 T_m



Taylor impact test: final length and mushroom diameter validate the model

Limits of ZA and tips for DEFORM

Known limits

- Valid roughly **below 0.5 T_m**: no dynamic recovery, recrystallisation, phase change
- BCC hardening is athermal → over-predicts hardening at high T
- Above ≈ 10^4 s^{-1} **dislocation drag** (viscous) dominates — not in the model
- Comparative study (Banerjee 2005, Cu): very good for room-T Taylor tests; **over-estimates the T-dependence** at high rates; larger errors at 1235 K
- Original constants come from one material condition — grain size and prior cold work change C_0

Checklist for DEFORM

- Choose the ZA entry in **Plastic → Flow stress**; constants a, c_1, c_3, c_4, c_5, n
- Constants are fitted with **T in kelvin** and stress in MPa — check against the problem's unit system before typing published values
- Use a **coupled thermal** simulation: adiabatic heating at high rates is essential
- Plot $\bar{\sigma}(\bar{\varepsilon})$ at 2–3 temperatures and rates in the material window and compare with the test data
- Hot forming above 0.6 T_m → prefer tabular data or sinh-Arrhenius

Zerilli–Armstrong in five points

1. Derived from **thermally activated dislocation motion**: $\bar{\sigma}$ = athermal + thermal + Hall–Petch
2. Thermal term $e^{-C_3T+C_4T\ln\dot{\varepsilon}}$ **couples** T and $\dot{\varepsilon}$
3. **BCC**: T, $\dot{\varepsilon}$ act on yield (A = const) · **FCC**: on hardening ($A \propto \bar{\varepsilon}^{-½}$) · HCP: both
4. Best for **high rates below 0.5 T_m**: impact, machining, high-speed forming
5. Modified ZA = semi-empirical fit for hot forming; DEFORM has the BCC form built in

Check yourself

1. What is the activation area, and how does it differ between BCC and FCC?
2. Which constant couples temperature and strain rate?
3. Why is ZA better than J–C for iron at 10^3 s^{-1} and −50 °C?
4. Why should you not use the original ZA for forging at 1150 °C?

Press ↑ to return to the main lecture, or → to continue directly with hot forming models.

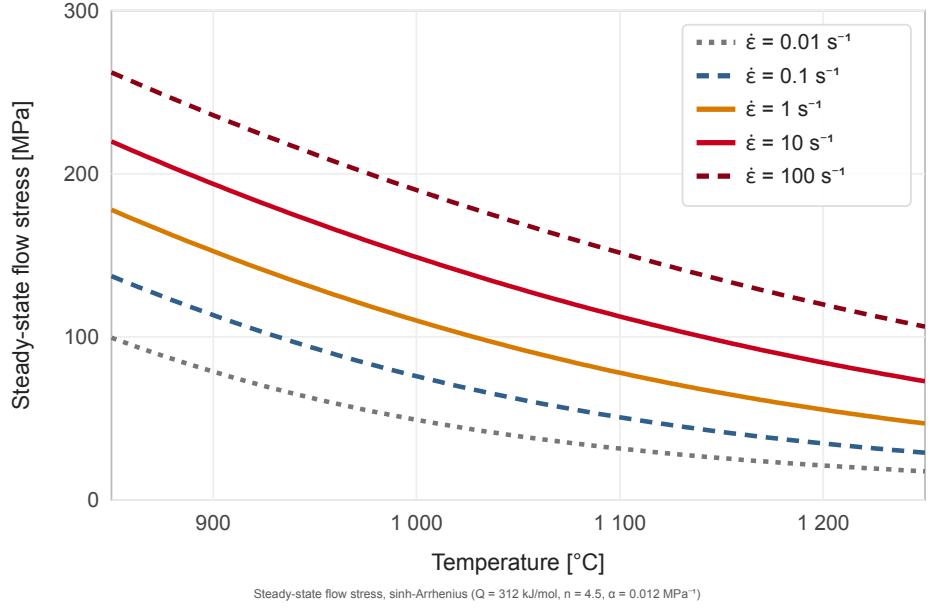
Hot forming: Zener–Hollomon, sinh-Arrhenius, Norton–Hoff

$$Z = \dot{\varepsilon}\,\exp\!\left(\frac{Q}{RT}\right) = A\,[\sinh(\alpha\bar{\sigma})]^n$$

$$\Rightarrow\ \bar{\sigma} = \frac{1}{\alpha}\,\operatorname{arsinh}\!\left[\left(\frac{Z}{A}\right)^{1/n}\right]$$

Norton–Hoff: $\bar{\sigma} = K\,(\sqrt{3})^{m+1}\,\dot{\bar{\varepsilon}}^{\,m},\quad K = K(T,\bar{\varepsilon})$

- One parameter Z combines T and $\dot{\varepsilon}$ — higher Z ⇒ higher stress, finer grain
- C–Mn steel: Q ≈ 300–320 kJ/mol; Al alloys ≈ 150–160 kJ/mol
- DEFORM: sinh form and power form $\dot{\bar{\varepsilon}} = A\bar{\sigma}^n e^{-\Delta H/RT}$ in the list



Steady-state flow stress, sinh-Arrhenius (Q = 312 kJ/mol, n = 4.5, α = 0.012 MPa⁻¹)

▼ Deep dive: 10 slides on Zener–Hollomon and the sinh law — press ↓

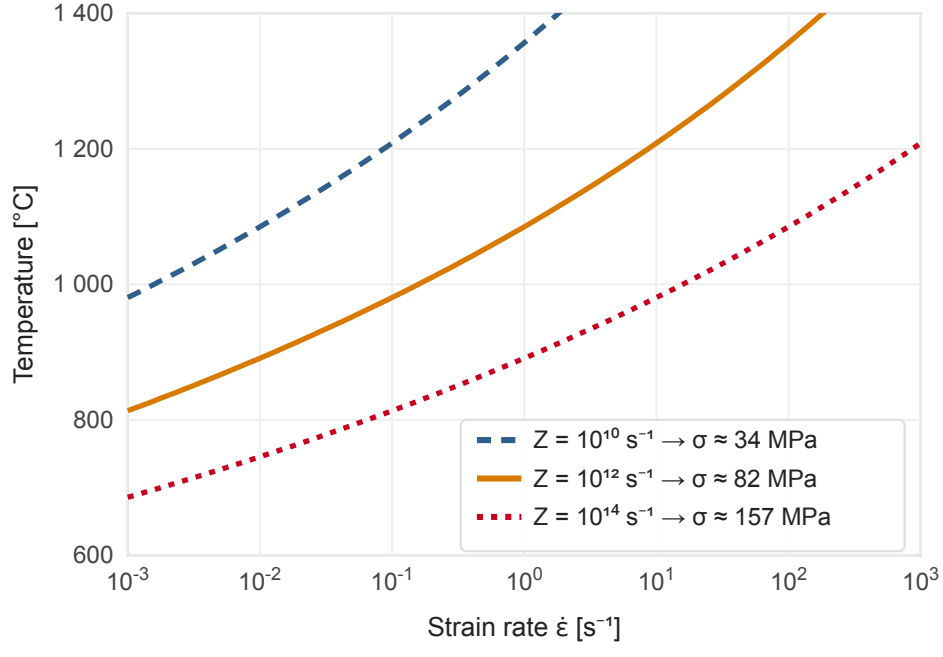
Zener–Hollomon ↓ 1 · Origin and idea

Temperature-compensated strain rate

- **C. Zener & J. H. Hollomon**, “Effect of strain rate upon plastic flow of steel”, *J. Appl. Phys.* 15 (1944) 22
- Observation: raising $\dot{\varepsilon}$ has the same effect on flow stress as **lowering T**
- Both enter through one parameter:

$$Z = \dot{\varepsilon}\,\exp\left(\frac{Q}{RT}\right) \quad [\mathrm{s}^{-1}]$$

- Q activation energy [J/mol], R = 8.314 J/(mol·K), **T absolute [K]**
- Same Z ⇒ same flow stress **and** similar microstructure (DRX grain size)
- Example (Q = 312 kJ/mol): 1000 °C, 1 s^{-1} → Z ≈ 6.3·10^{12} s^{-1}; 900 °C, 10 s^{-1} → 7.8·10^{14}; 1200 °C, 0.01 s^{-1} → 1.2·10^{9}



Lines of constant Z: a lab test at low $\dot{\varepsilon}$ can stand in for a fast press at higher T (C–Mn steel)

Zener–Hollomon ↓ 2 · Physical basis

Why Arrhenius? Hot working as fast high-temperature creep

- Above ≈ 0.5 T_m plastic flow is controlled by **dislocation climb and cross-slip** — both need vacancy diffusion
- Diffusion is thermally activated: $D = D_0\, e^{-Q_D/RT}$ → strain rate inherits the Arrhenius term
- Hardening (dislocation multiplication) and softening (dynamic recovery, **DRX**) balance → steady state
- Q of hot working ≥ Q of self-diffusion: alloying, solute drag and particles raise the *apparent* Q

$$\dot{\varepsilon} = f(\bar{\sigma})\, \exp\left(-\frac{Q}{RT}\right) \iff Z = f(\bar{\sigma})$$

Material	Apparent Q for hot working [kJ/mol]
Pure Al (self-diffusion ≈ 142)	≈ 140 – 160
Al alloys (6xxx, 7xxx)	≈ 140 – 200
Mg alloy AZ31	≈ 130 – 170
C–Mn and low-alloy steels (30CrMoA: 262)	≈ 260 – 320
Austenitic stainless steels	≈ 400 – 460
Ti-6Al-4V: β / (α + β) field	≈ 246 / 621
Ni-base superalloys	≈ 400 – 600

Typical literature ranges; 30CrMoA from Zhang et al. 2020; Ti-6Al-4V from "Hot deformation characteristics of Ti-6Al-4V" (CityU)

Power law, exponential law — and the sinh law that joins them

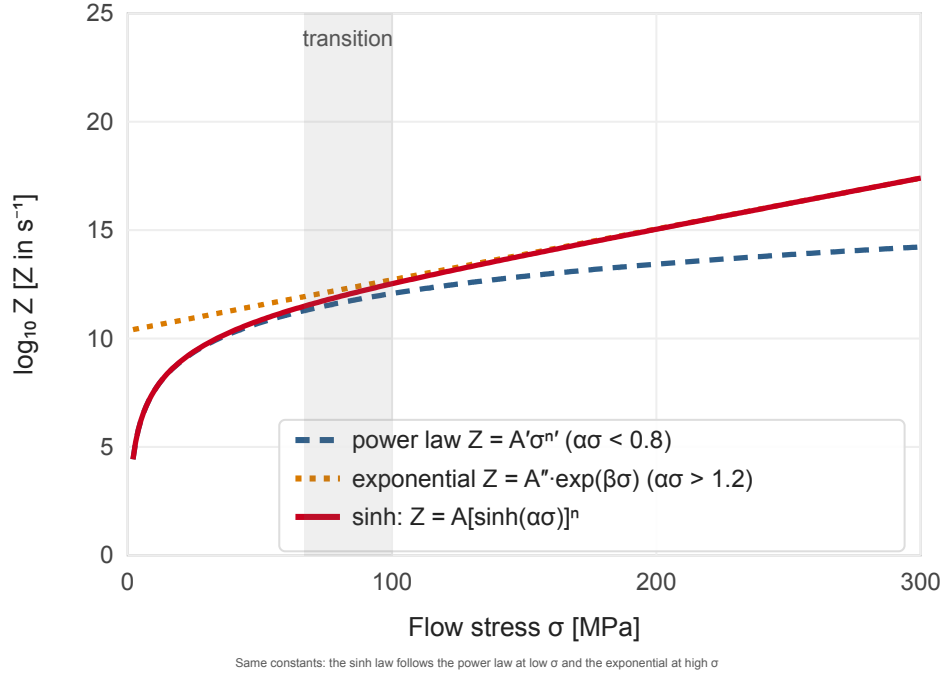
Low stress (ασ < 0.8): $Z = A_1\,\bar{\sigma}^{\,n'}$

High stress (ασ > 1.2): $Z = A_2\,e^{\beta\bar{\sigma}}$

All stresses (Garofalo 1963; Sellars & Tegart 1966):

$$Z = A\,[\sinh(\alpha\bar{\sigma})]^n, \qquad \alpha \approx \beta/n'$$

- sinh x ≈ x for small x → power law; sinh x ≈ $e^x/2$ for large x → exponential
- Typical: n ≈ 3–6, α ≈ 0.01–0.04 MPa^{-1} (steels ≈ 0.01, Al alloys larger)
- Power law = creep (“power-law breakdown” at high Z); exponential = high-rate hot working



Same constants: the sinh law follows the power law at low σ and the exponential at high σ

Inverting the law and what follows from it

Flow stress as a function of Z

$$\bar{\sigma} = \frac{1}{\alpha} \ln\left\{ \left(\frac{Z}{A}\right)^{1/n} + \left[\left(\frac{Z}{A}\right)^{2/n} + 1 \right]^{1/2} \right\}$$

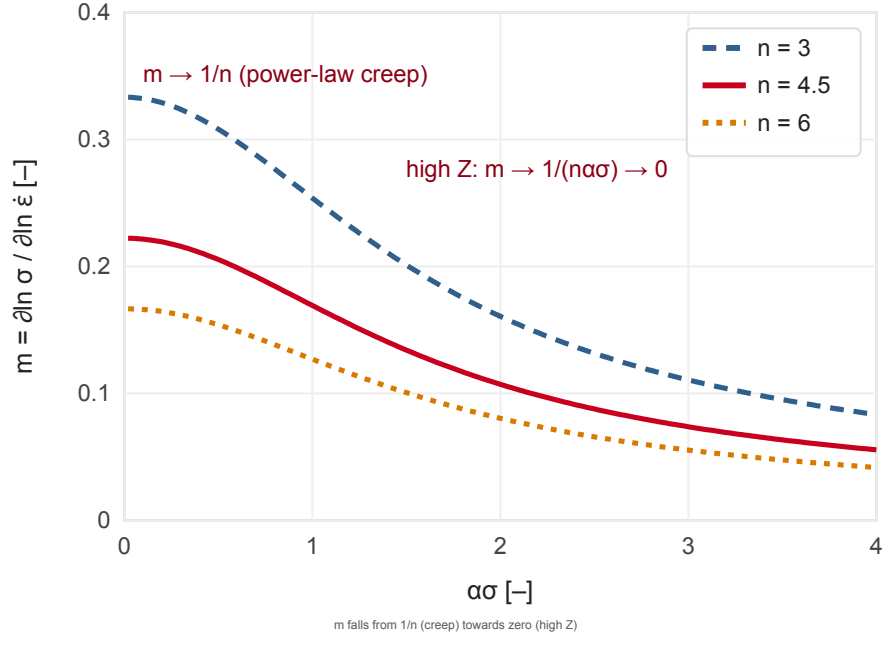
Strain-rate sensitivity

$$m = \frac{\partial \ln \bar{\sigma}}{\partial \ln \dot{\varepsilon}} = \frac{\tanh(\alpha \bar{\sigma})}{n\, \alpha \bar{\sigma}}$$

Temperature sensitivity

$$\frac{\partial \ln \bar{\sigma}}{\partial T} = -\, m\, \frac{Q}{R T^2}$$

- Example: 1000 °C, 1 s^{-1} → σ = 110 MPa, ασ = 1.32, m ≈ 0.15; −10 K ≈ +3 % stress



Determining α, n, Q and A from hot compression tests

1 · Tests: Gleeble compression, 4–6 T × 4 $\dot{\varepsilon}$ (e.g. 900–1200 °C, 0.01–10 s^{-1}); peak or steady-state σ

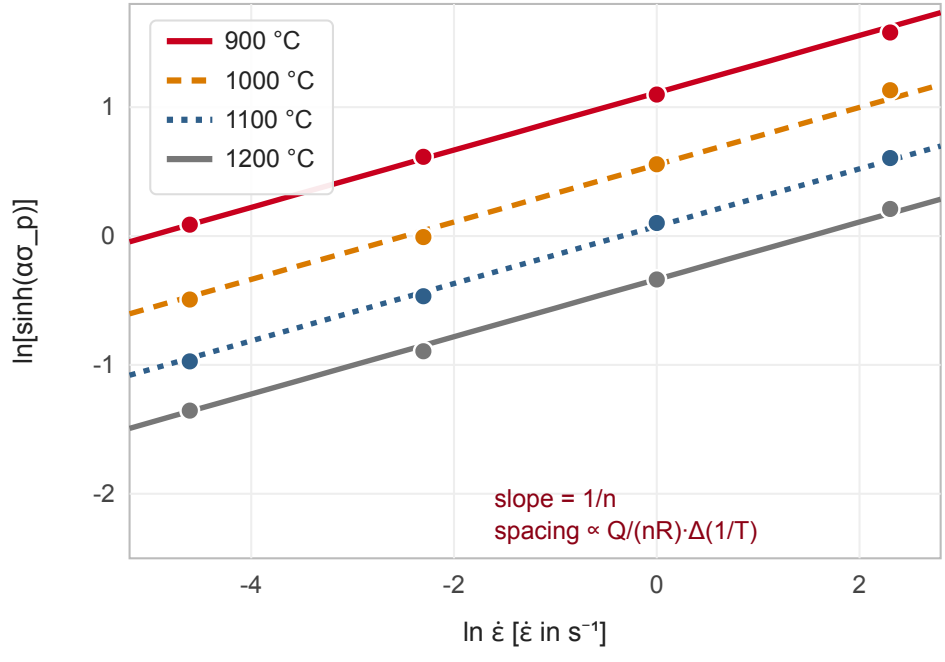
2 · n′ and β: slope of ln $\dot{\varepsilon}$ vs ln σ (low σ) and ln $\dot{\varepsilon}$ vs σ (high σ) → α = β/n′

3 · n: $\ln \dot{\varepsilon} = \ln A - \frac{Q}{RT} + n \ln \sinh(\alpha\sigma)$ → **slope of ln sinh vs ln $\dot{\varepsilon}$ = 1/n**

4 · Q = n R × slope of ln sinh(ασ) vs 1/T (at constant $\dot{\varepsilon}$)

5 · A from the intercept of ln Z vs ln sinh(ασ); check with R and AARE

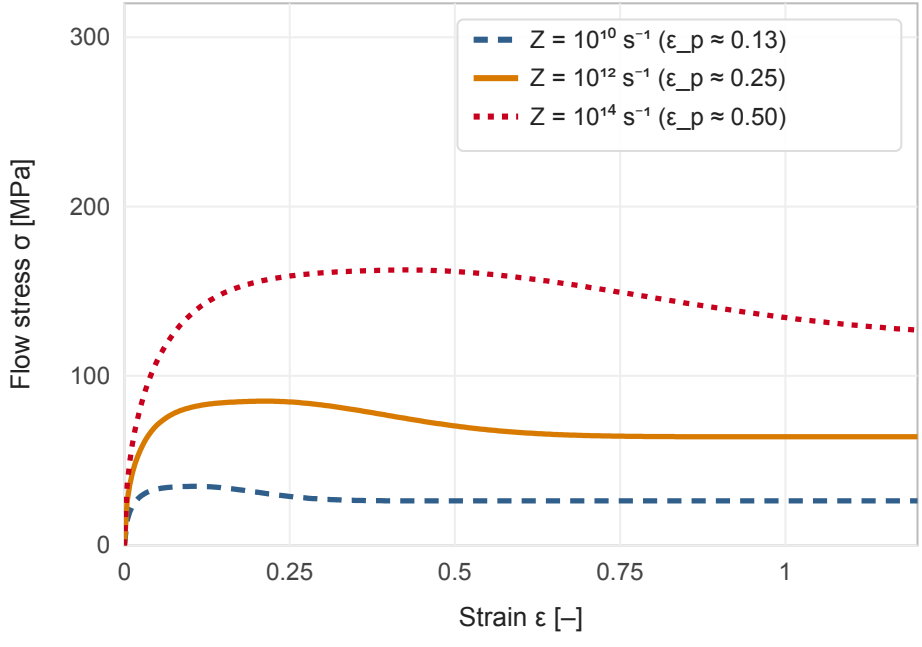
Example 30CrMoA (Zhang et al. 2020): α = 0.011 MPa^{-1}, n = 4.40, Q = 262 kJ/mol, A = 3.67·10^9 s^{-1}, R = 0.983



Step 3: parallel lines, one per temperature (synthetic points from the C–Mn constants)

Zener–Hollomon ↓ 6 · Whole flow curves

From one stress value to the whole flow curve



Typical DRX-type curves: peak, then softening to steady state (illustrative, σ_p from the sinh law)

Strain-compensated Arrhenius model

$\alpha, n, Q, \ln A = f(\bar{\varepsilon})$: polynomials of 4th–6th order, fitted at ε = 0.05 … 0.8

- Constants solved at each strain → smooth polynomial → full curve σ(ε, ε̇, T)
- Widely used since Lin et al. 2008 (42CrMo); accuracy typically R ≈ 0.98–0.99, AARE ≈ 3–6 %
- Downside: 20–30 coefficients, no physics in the polynomials, poor outside the fitted strain range

Physics of the curve

- Peak stress σ_p and steady state σ_{ss} both rise with Z
- Peak strain grows with Z and d_0: $\varepsilon_p = a\, d_0^{h}\, Z^{c}$, DRX starts at $\varepsilon_c \approx 0.8\, \varepsilon_p$

Z controls grain size — and the safe processing window

DRX grain size: $d_{DRX} = a\,Z^{-m_d}$, $m_d \approx 0.1–0.3$

Steady-state stress ↔ grain size: $\sigma_{ss} \propto d_{DRX}^{-k}$, $k \approx 0.7–0.8$

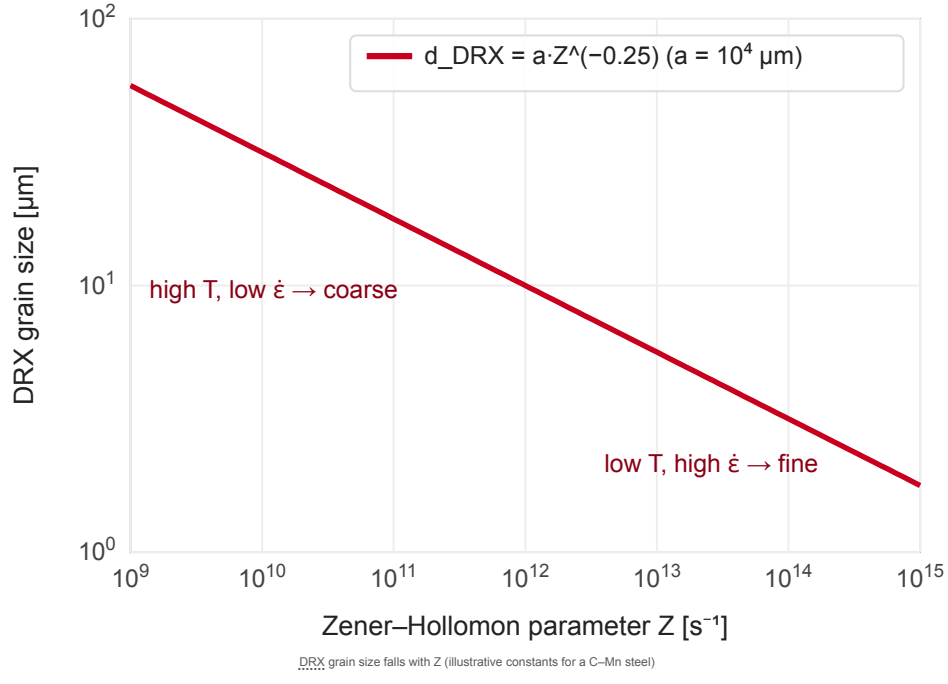
- Same equations sit in DEFORM's **Grain** data (JMAK, block 8)
- Finish forging at **higher Z** (lower T, faster) → finer grain, higher strength

Processing maps (Prasad, dynamic materials model)

Efficiency: $\eta = \dfrac{2m}{m+1}$

Instability: $\xi = \dfrac{\partial \ln[m/(m+1)]}{\partial \ln \dot{\varepsilon}} + m < 0$

- DRX domains: η ≈ 30–55 % (pure Al: max ≈ 55 %) · instability: shear bands, cracking



DRX grain size falls with Z (illustrative constants for a C–Mn steel)

Norton–Hoff, Hensel–Spittel — and how DEFORM takes the data

Norton–Hoff (viscoplastic power law)

$$\bar{\sigma} = K(T,\bar{\varepsilon})\,(\sqrt{3})^{m+1}\,\dot{\bar{\varepsilon}}^{\,m}$$ — e.g. FORGE; m ≈ 0.1–0.2 hot

Hensel–Spittel (empirical, many terms)

$$\bar{\sigma} = A\,e^{m_1 T}\,\bar{\varepsilon}^{\,m_2}\,\dot{\bar{\varepsilon}}^{\,m_3}\,e^{m_4/\bar{\varepsilon}}\,(1+\bar{\varepsilon})^{m_5 T}\,e^{m_7\bar{\varepsilon}}\,\dot{\bar{\varepsilon}}^{\,m_8 T}\,T^{m_9}$$

- Common in FORGE, QForm, Simufact libraries — no Z, but the same data source (hot compression)

DEFORM flow-stress list

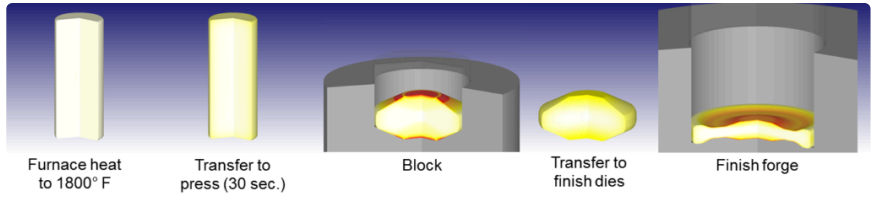
sinh: $\dot{\bar{\varepsilon}} = A\,[\sinh(\alpha\bar{\sigma})]^n \exp[-\Delta H/(R\,T_{abs})]$

power: $\dot{\bar{\varepsilon}} = A\,\bar{\sigma}^{\,n} \exp[-\Delta H/(R\,T_{abs})]$

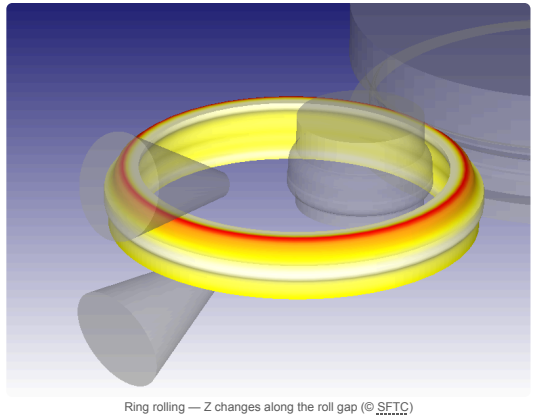
- ΔH ≡ Q; DEFORM uses **absolute** temperature — check the units it expects for ΔH and R in your unit system
- Alternative: compute σ(ε, ε̇, T) from the law and enter it as a **table** — easy to check, and strain hardening at small ε can be added
- Plot the curves in the material window at the process T and ε̇ before running

Where Z and the sinh law are used

Operation	Use of Z / sinh law
Hot closed-die forging	Flow stress for die fill and load; Z → DRX grain size in the part
Hot rolling, pass schedules	Mean flow stress per pass; Z → recrystallisation between passes
Aluminium extrusion	Sinh law is the classic model (Sheppard & Wright); exit temperature and speed limits
Isothermal / hot-die forging of Ti, Ni alloys	Low $\dot{\varepsilon}$, near-creep regime; m from the sinh law
Process design and scale-up	Equal Z: transfer Gleeble results to the press
Processing maps	Choose safe T–$\dot{\varepsilon}$ windows (DRX, no shear bands)
Creep of dies and hot tools	Same Arrhenius structure, low-stress (power-law) branch



Hot forging sequence of a disc (© SFTC)



Ring rolling — Z changes along the roll gap (© SFTC)

Limits, summary and questions

Limits

- Steady-state (or peak) law — needs strain compensation or a DRX model for the full curve
- Q, n, α valid only in the tested T–ε̇ window; **phase changes** (α/γ, α/β) change Q
- Not for cold or warm forming: below ≈ 0.5 T_m use power law or J–C
- Adiabatic heating at high ε̇ distorts test curves → correct before fitting

Summary

1. $Z = \dot{\varepsilon} \cdot \exp(Q/RT)$: one parameter for T and ε̇ (T in K)
2. Physics: thermally activated, diffusion-controlled flow
3. Sinh law joins power and exponential laws; $\sigma = (1/\alpha) \cdot \operatorname{arsinh}[(Z/A)^{1/n}]$
4. Z also sets DRX grain size and processing windows

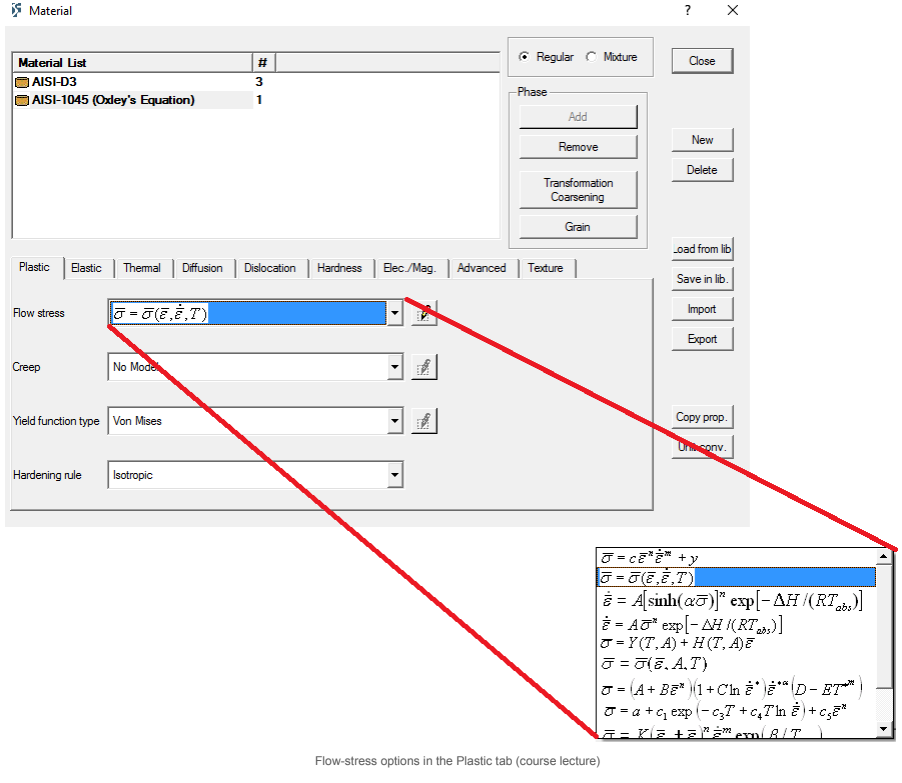
Check yourself

1. Compute Z for 1100 °C, 5 s^{-1} and Q = 300 kJ/mol.
2. Why does the sinh law work at low and at high stresses?
3. Which plot gives n, and which gives Q?
4. A forging should get a finer grain. Raise or lower Z — and how in practice?
5. Why is a strain-compensated model risky beyond ε = 0.8?

Press ↑ to return to the main lecture, or → to continue directly.

Flow stress Beyond equations

Microstructure-based, AI and user-defined flow stress



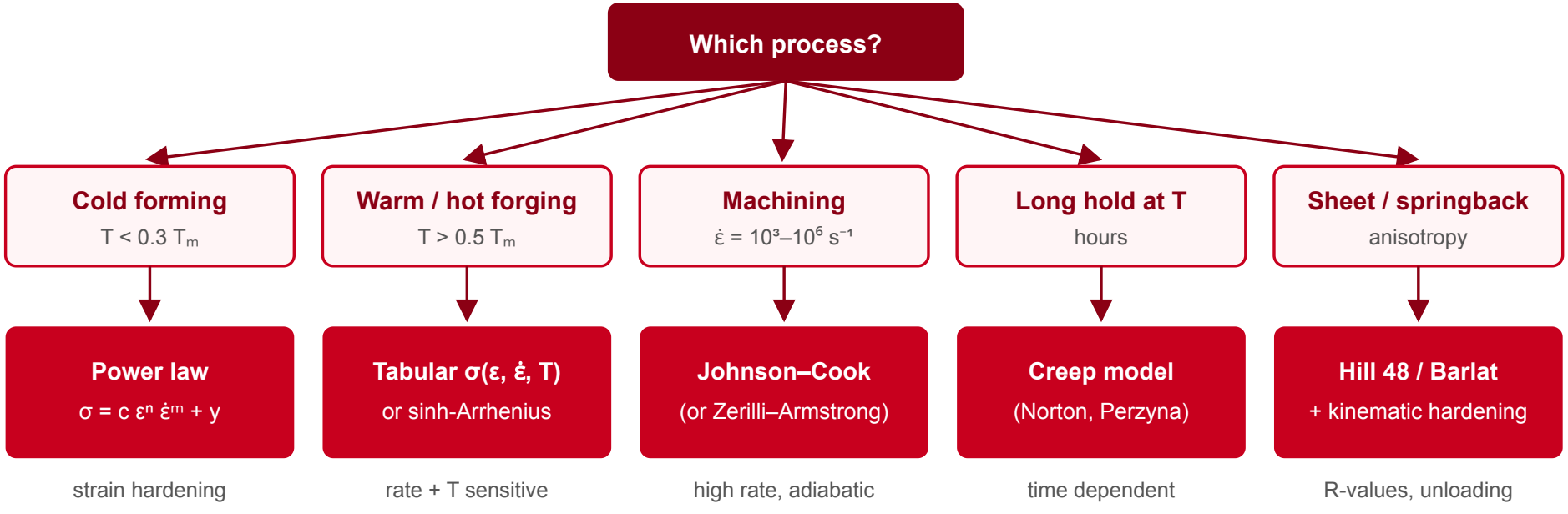
Flow-stress options in the Plastic tab (course lecture)

Option	Idea	When
Dislocation density	$\bar{\sigma} = \sigma_0 + \alpha M G b\sqrt{\rho}$; ρ evolves with ε, $\dot{\varepsilon}$, T	Multi-step hot forging, DRX
Texture / crystal plasticity	Grain orientations → anisotropic flow	Ti, Mg, sheet, earing
DNN predictor	Neural network: chemistry → flow curve	No test data, early design
User routine	Own FORTRAN subroutine compiled into the solver	Research models
Conversion	Equation → table; test data → fitted model	Library building

The **Material Suite** (SFTC) fits flow stress from compression data, computes TTT diagrams from composition and JMAK constants.

Flow stress Choosing a model

Match the model to the process

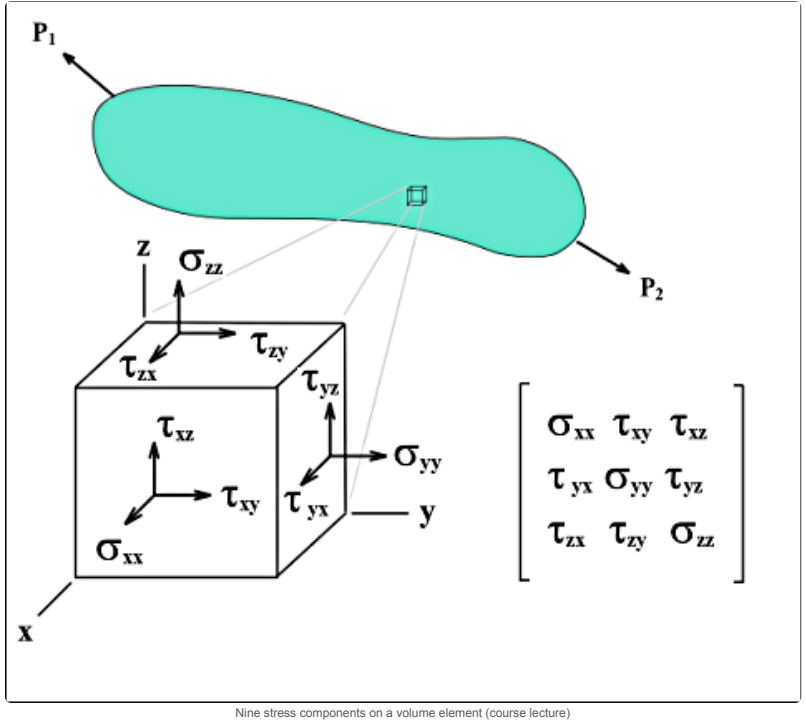


Always: check the measured ε, ε̇ and T range against the process window

Decision guide used in this course

Yield & hardening Effective stress

From the stress tensor to one number: effective stress



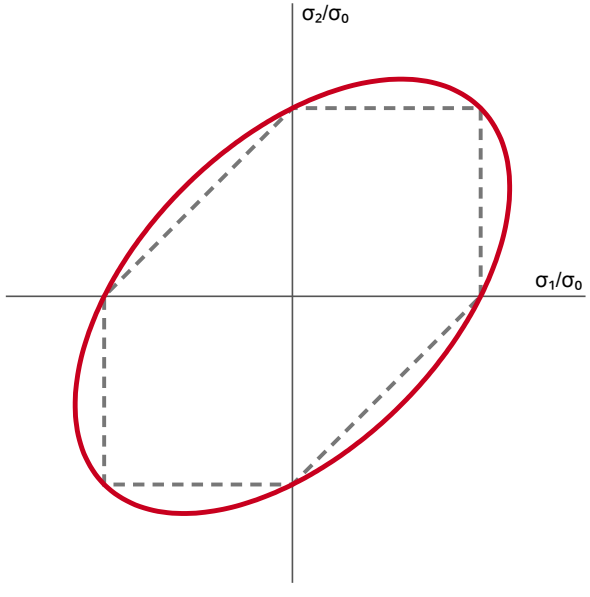
Nine stress components on a volume element (course lecture)

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}, \quad s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$$

$$\bar{\sigma} = \sqrt{\tfrac{3}{2}\, s_{ij} s_{ij}} = \sqrt{\tfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

- Hydrostatic part σ_m does **not** cause yielding of dense metals
- Flow curve (uniaxial) is compared with $\bar{\sigma}$ → yield when $\bar{\sigma} = \sigma_f(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$
- The **yield function** defines how $\bar{\sigma}$ is computed — Plastic tab, “Yield function type”

von Mises vs Tresca — isotropic yield criteria

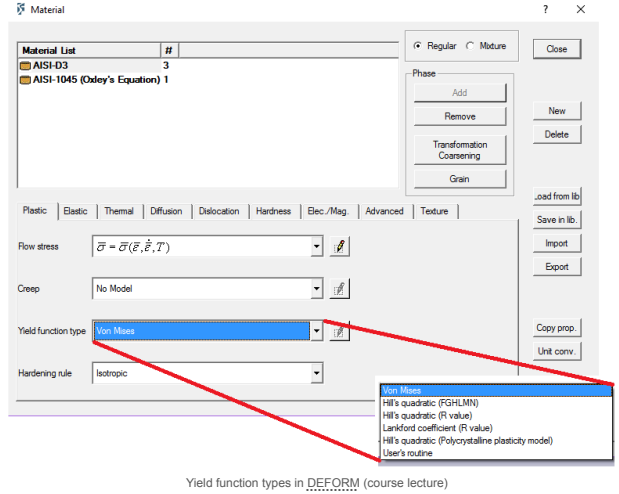


Plane-stress yield loci ($\sigma_3 = 0$)

Tresca: $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_0}{2}$

von Mises: $\bar{\sigma} = \sigma_0$

- Max. difference: **15.5 %** in pure shear ($k = \sigma_0/\sqrt{3}$ vs $\sigma_0/2$)
- Experiments on most metals lie closer to **von Mises**
- von Mises is smooth → convenient for FEM; DEFORM default



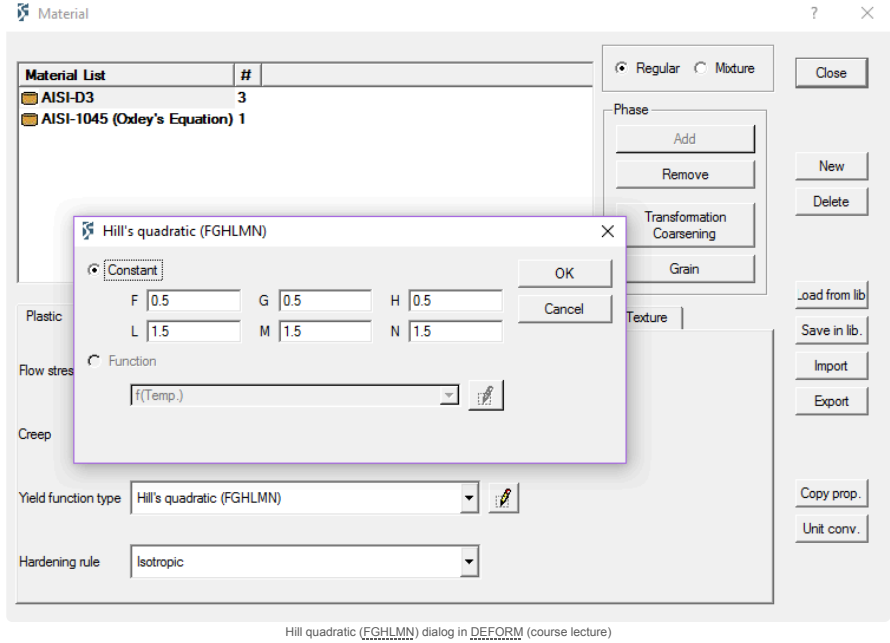
Yield function types in DEFORM (course lecture)

Hill 1948 — orthotropic anisotropy

$$F(\sigma_{yy}-\sigma_{zz})^2+G(\sigma_{zz}-\sigma_{xx})^2+H(\sigma_{xx}-\sigma_{yy})^2+2L\tau_{yz}^2+2M\tau_{zx}^2+2N\tau_{xy}^2=1$$

- 6 constants F, G, H, L, M, N; axes = rolling (x), transverse (y), normal (z)
- F = G = H = 1/(2σ₀²), L = M = N = 3/(2σ₀²) → von Mises
- DEFORM: enter **FGHLMN** directly or **R-values** (next slide)

From R-values (σ₀ in RD): $\frac{H}{G}=r_0,\quad \frac{H}{F}=\frac{r_0}{r_{90}},\quad \frac{N}{G}=\left(r_{45}+\frac{1}{2}\right)\left(1+\frac{r_0}{r_{90}}\right)$

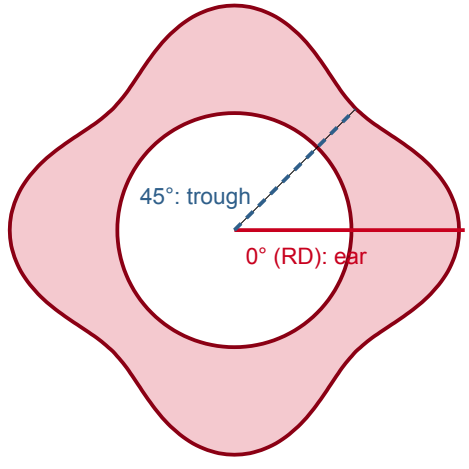


Hill quadratic (FGHLMN) dialog in DEFORM (course lecture)

Lankford R-values, earing and Barlat 91

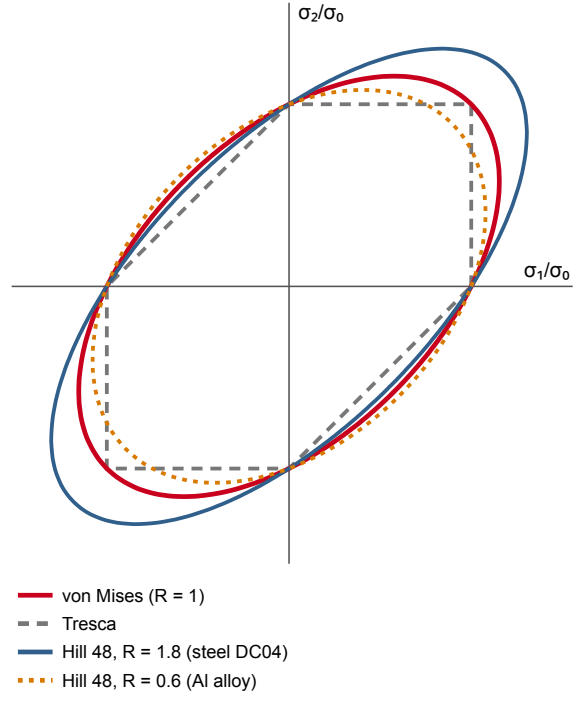
$$r = \frac{\varepsilon_w}{\varepsilon_t}, \quad \bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4}, \quad \Delta r = \frac{r_0 + r_{90} - 2r_{45}}{2}$$

- High $\bar{r}$ → good deep-drawability (DC04: $\bar{r} \approx 1.6$–1.9)
- $\Delta r \neq 0$ → **ears** on drawn cups
- Al alloys: r < 1 → Hill 48 fails (“anomalous” behaviour)
- **Barlat 91**: non-quadratic, exponent a = 6 (BCC) / 8 (FCC) — better for Al



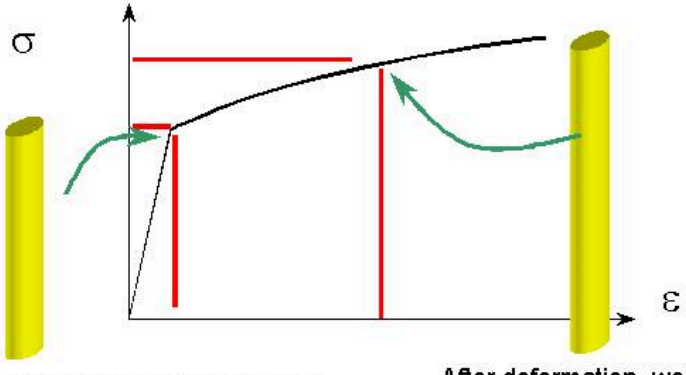
Cup top view, $\Delta r = (r_0 + r_{90} - 2r_{45})/2 > 0$

Four ears from planar anisotropy

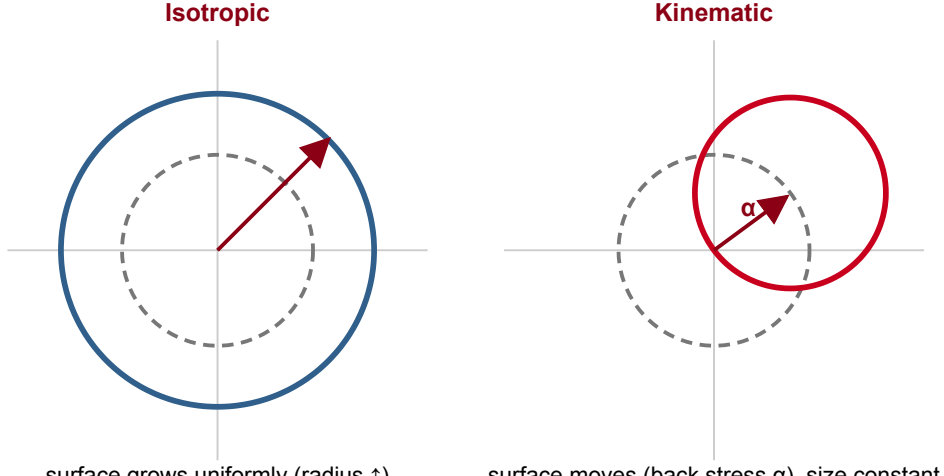


Hill 48 normal anisotropy, plane stress

Isotropic hardening — the yield surface expands



Work hardening: reloading needs a higher stress (course lecture)

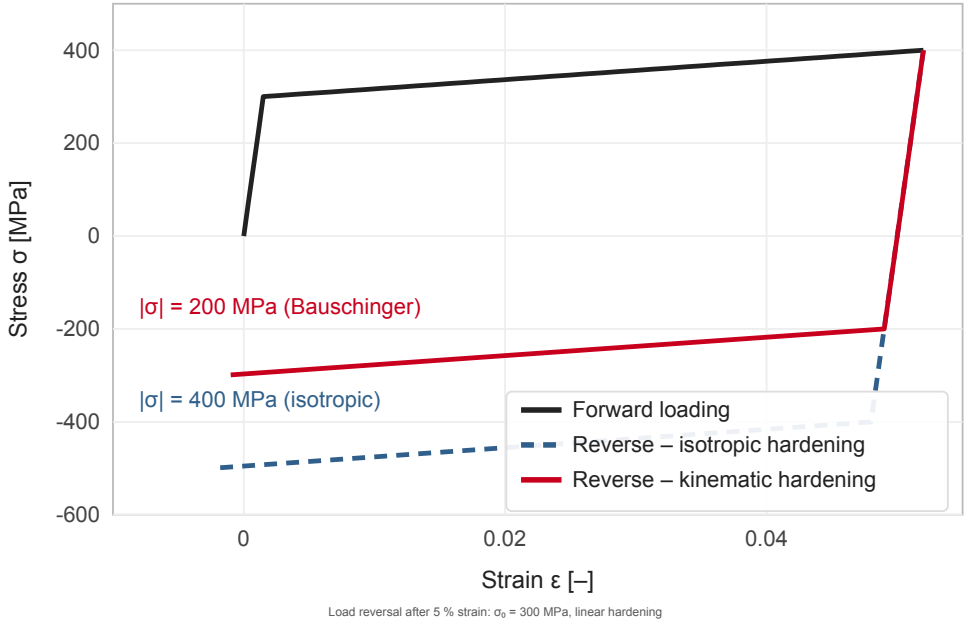


Isotropic (left) and kinematic (right) hardening in the deviatoric plane

Isotropic: $f = \bar{\sigma}(\boldsymbol{\sigma}) - \sigma_f(\bar{\varepsilon}^p) = 0$ — surface radius grows with $\bar{\varepsilon}^p$

- Default in DEFORM; exact for **monotonic** loading (most forging)

Kinematic hardening — the yield surface translates



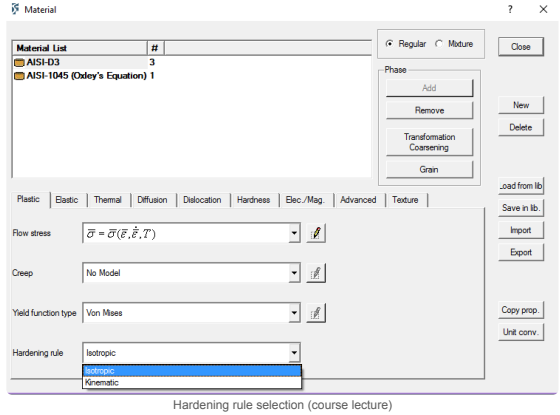
Load reversal after 5 % strain: σ_0 = 300 MPa, linear hardening

$$f = \bar{\sigma}(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \sigma_0 = 0$$

Prager / Ziegler: $\dot{\boldsymbol{\alpha}} = c\,\dot{\boldsymbol{\varepsilon}}^p$

Armstrong–Frederick: $\dot{\boldsymbol{\alpha}} = c\,\dot{\boldsymbol{\varepsilon}}^p - \gamma\,\boldsymbol{\alpha}\,\dot{\bar{\varepsilon}}^p$

- **Bauschinger effect**: after tension the material yields earlier in compression
- Matters for **bending–unbending** over die radii and drawbeads, cyclic loading
- Mixed hardening: fraction isotropic + fraction kinematic
- Data: tension–compression or cyclic shear tests

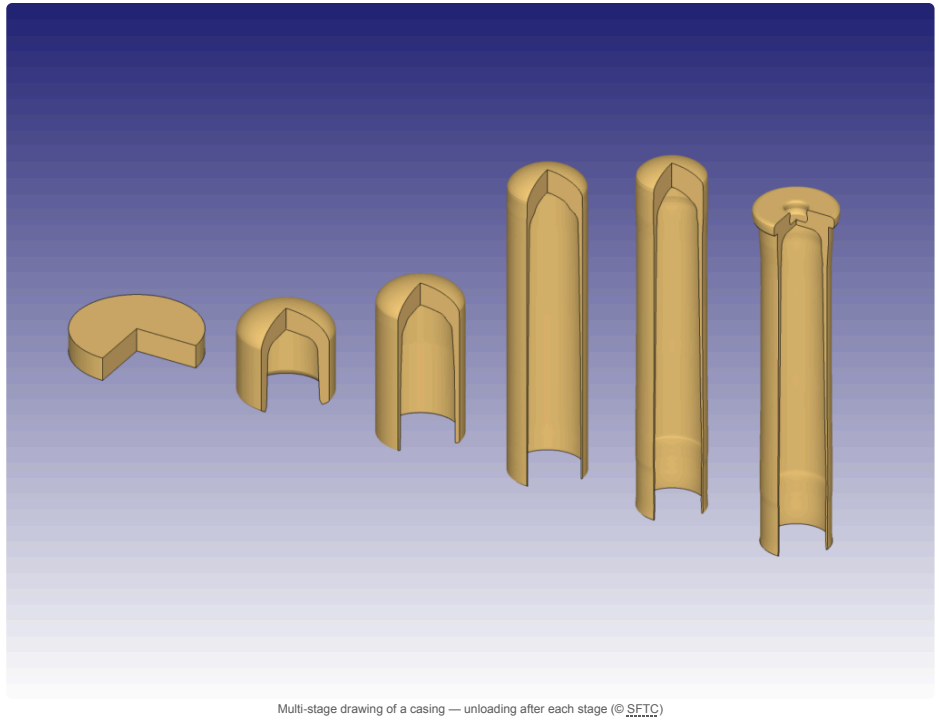


Hardening rule selection (course lecture)

Yoshida–Uemori two-surface model and springback

- **Two surfaces**: inner yield surface moves inside a bounding surface
- Captures: early re-yielding, **transient** Bauschinger, permanent softening, work-hardening stagnation
- Plus **E-modulus degradation** with plastic strain: $E = E_0 - (E_0 - E_a)\,[1 - e^{-\xi \bar{\varepsilon}^p}]$
- 7 material constants (Y, C, B, R_{sat}, b, m, h) from cyclic tension–compression tests
- Standard for **AHSS** springback (DP, TRIP, CP steels)

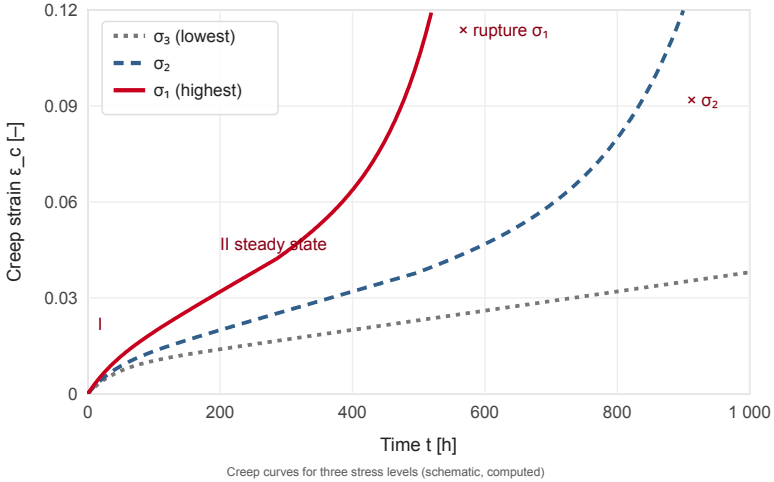
> Springback check list: elasto-plastic object · E($\bar{\varepsilon}^p$) · kinematic / Y-U hardening · anisotropic yield · ≥ 5 elements through thickness



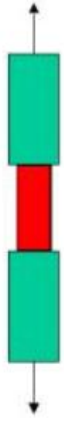
Multi-stage drawing of a casing — unloading after each stage (© SFTC)

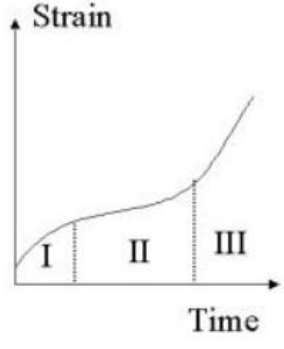
Creep Basics

Creep: plastic flow at constant stress over time

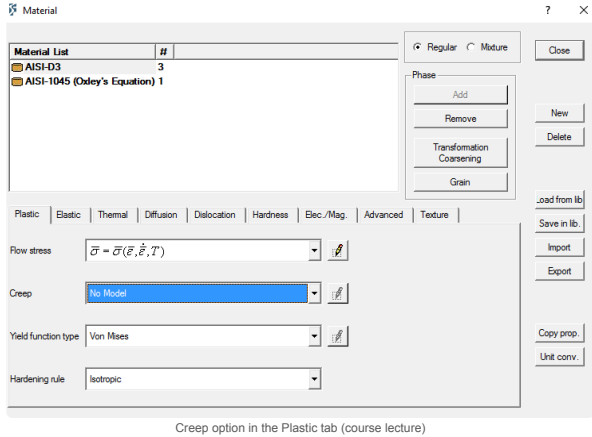


Creep curves for three stress levels (schematic, computed)





Stages I, II, III (course lecture)



Creep option in the Plastic tab (course lecture)

- Significant above ≈ **0.4 T_m**; strain grows at constant load — I primary, II steady (minimum rate), III tertiary → rupture
- In DEFORM creep is added to the elasto-plastic strain: $\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^c + \dot{\varepsilon}^{th}$

Creep Models

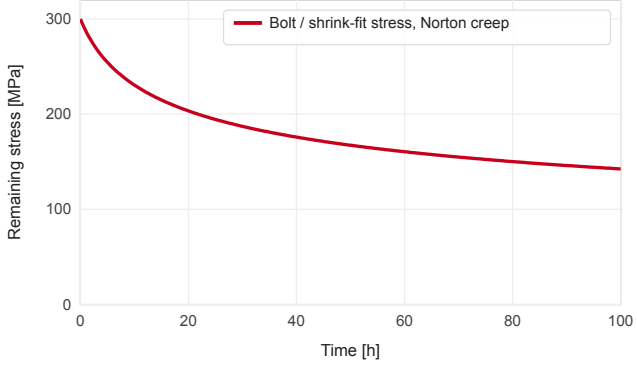
Creep models available in DEFORM

Model	Equation	Typical use
Power law (Norton)	$\dot{\varepsilon}^c = A\,\bar{\sigma}^n$	Steady-state creep, relaxation
Bailey–Norton	$\varepsilon^c = A\,\bar{\sigma}^n\,t^m$ (m < 1)	Primary + secondary creep
Perzyna	$\dot{\varepsilon}^c = \gamma \left\langle \frac{\bar{\sigma}}{\sigma_y} - 1 \right\rangle^p$	Viscoplasticity above yield
Soderberg	$\dot{\varepsilon}^c = A\,[\exp(\bar{\sigma}/\sigma_0) - 1]$	Low-stress creep
Exponential	$\dot{\varepsilon}^c = A\,e^{B\bar{\sigma}}\,e^{-Q/RT}$	High-stress creep, T-dependent
Tabular	$\dot{\varepsilon}^c = f(\bar{\sigma}, T, \varepsilon^c)$	Measured data

- Norton exponent n: 3–8 for dislocation creep; ≈ 1 for diffusion creep
- Fit constants to the **stress and temperature range of the process** — creep data extrapolate even worse than flow stress

Model list: DEFORM V14 manual, 10 Material Data – Creep

Where creep shows up in forming



Stress relaxation under fixed strain, Norton n = 5 (computed)

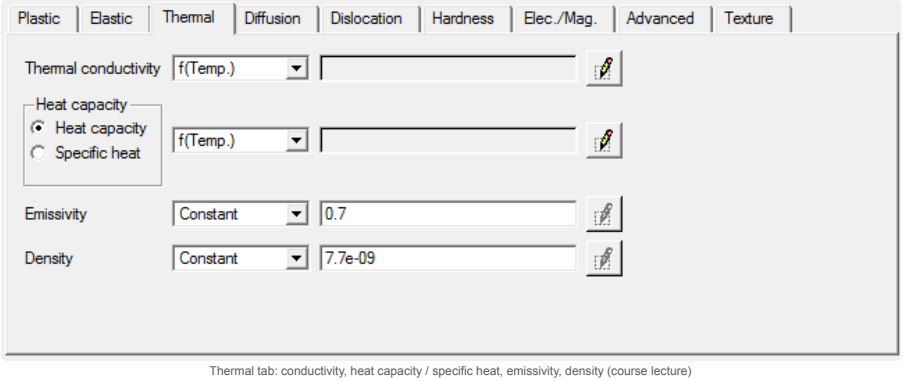
- **Stress relaxation**: shrink-fit die rings and bolts at temperature lose preload
- **Isothermal / hot-die forging** of Ti and Ni alloys at low $\dot{\varepsilon}$ (10^{-3}–10^{-2} s^{-1})
- **Superplastic forming** of Ti-6Al-4V, Al 5083 (m ≥ 0.5)
- **Hot sizing, stress-relief annealing**: residual stress drops during holding
- Heat treatment: creep during slow cooling and tempering

Picture not available offline:
Dead weight creep testing frame
https://www.doitpoms.ac.uk/tlplib/creep/dead_weight_frame.png

Dead-weight creep frame (© DoITPoMS, Univ. of Cambridge)

Thermal data Thermal tab

The Thermal tab



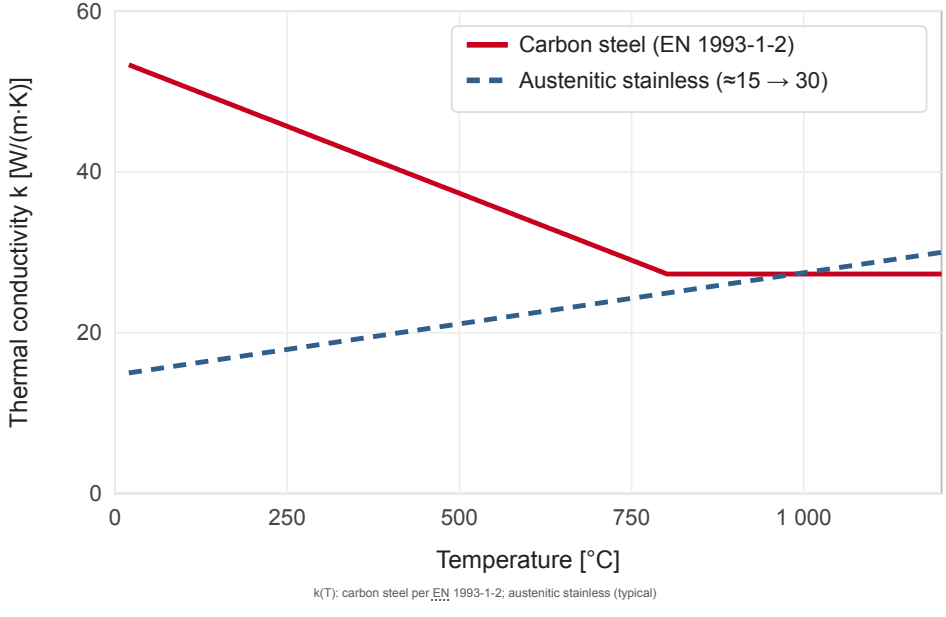
Thermal tab: conductivity, heat capacity / specific heat, emissivity, density (course lecture)

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \beta \bar{\sigma} \dot{\bar{\varepsilon}} + \dot{q}_{tr} + \dot{q}_{EM}$$

Property	SI unit in DEFORM	Controls
Conductivity k(T)	N/(s·°C) = W/(m·K)	Heat flow inside the part
Heat capacity $\rho c_p(T)$	N/(mm²·°C)	Heating per unit energy
Emissivity ε	–	Radiation to environment
Density ρ	t/mm³	Mass (hammers), specific-heat option

Thermal data Conductivity

Thermal conductivity k(T)



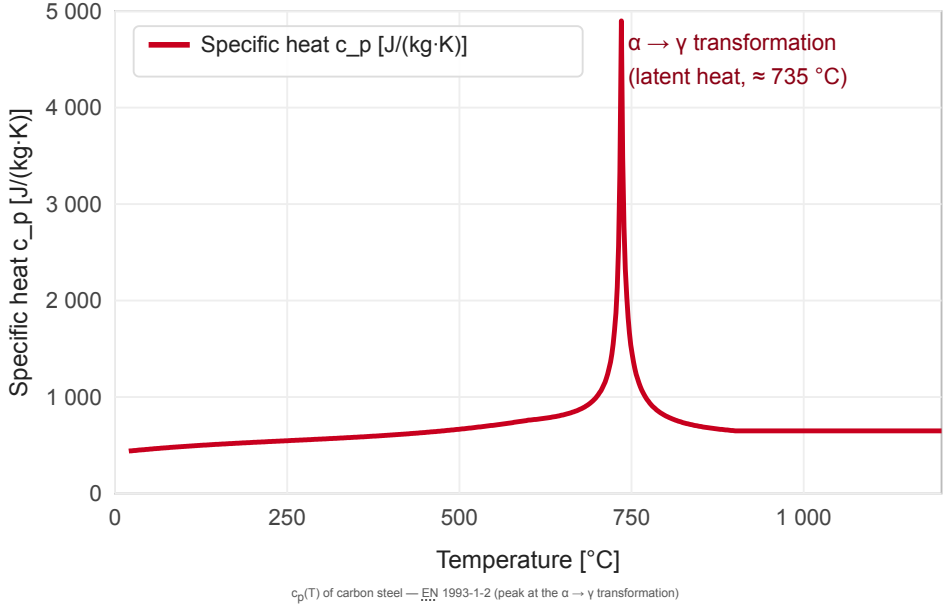
k(T): carbon steel per EN 1993-1-2; austenitic stainless (typical)

Material, 20 °C	k [W/(m·K)]
Copper	≈ 390
Aluminium alloys	120 – 200
Carbon steel	45 – 54
Hot-work tool steel H13	≈ 25 – 29
Stainless 304	≈ 15
Ti-6Al-4V	≈ 7

- Low k (Ti, Ni, stainless) → **chilled skin** at die contact, local overheating in shear bands
- Carbon steel: k halves from 20 °C to 800 °C — always *f(Temp)* for hot forging

Thermal data Heat capacity

Heat capacity or specific heat?



c_p(T) of carbon steel — EN 1993-1-2 (peak at the α → γ transformation)

$$\underbrace{\rho\, c_p}_{\text{heat capacity}} \quad [\mathrm{N/(mm^2\,^\circ C)}] = \rho\,[\mathrm{t/mm^3}] \times \underbrace{c_p}_{\text{specific heat}} \quad [\mathrm{mm^2/(s^2\,^\circ C)}]$$

- Steel at 20 °C: 7850 kg/m³ × 440 J/(kg·K) ≈ **3.45 N/(mm²·°C)**
- Above 900 °C: c_p ≈ 650 J/(kg·K) → ≈ **5.1 N/(mm²·°C)**
- Option *Specific heat* → DEFORM multiplies by density
- Latent heat of transformation: in the peak (Regular) *or* in Transformation data (Mixture) — **not both**

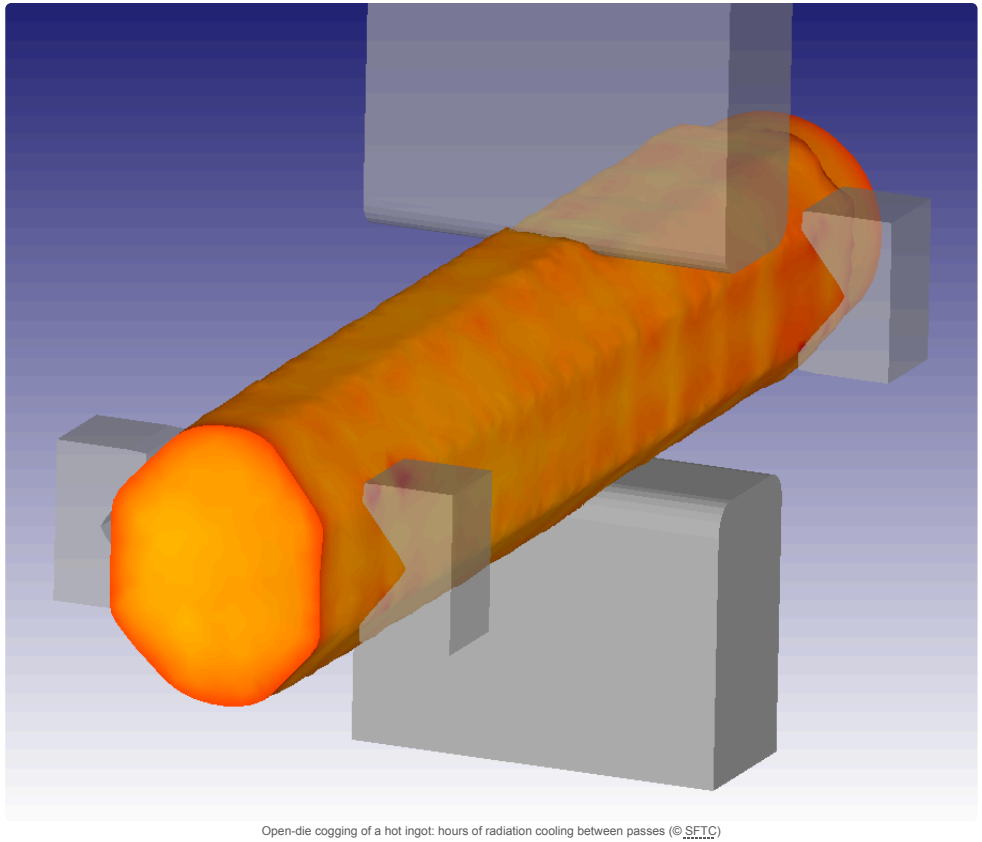
Emissivity and density

Radiation + convection at a free surface:

$$q = \varepsilon\,\sigma_{SB}\,(T^4 - T_\infty^4) + h_{conv}(T - T_\infty)$$

Surface	Emissivity ε
Oxidised / scaled steel, hot	0.7 – 0.9
Bright machined steel	0.2 – 0.4
Polished aluminium	0.04 – 0.1
Oxidised titanium	0.5 – 0.7

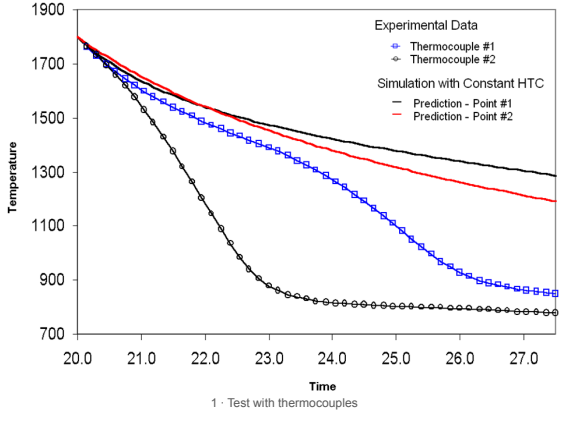
- At 1200 °C radiation dominates heat loss in transfer from furnace to press
- **Density**: mass for hammers and screw presses, specific-heat option, volume change ρ(T) in heat treatment



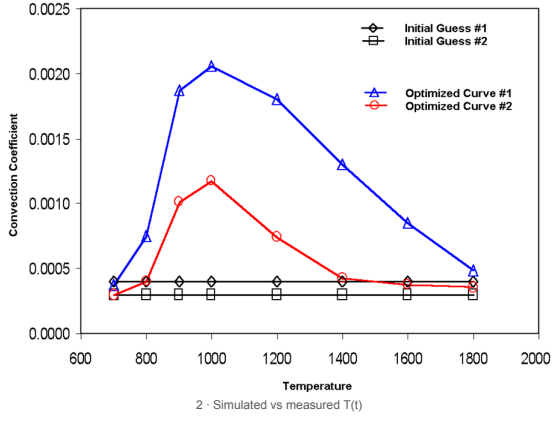
Open-die cogging of a hot ingot: hours of radiation cooling between passes (© SFTC)

Thermal data Interface HTC

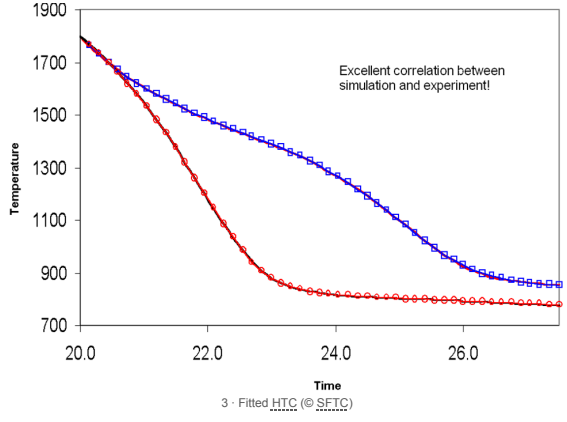
Die–workpiece heat transfer and the Inverse HTC module



1 · Test with thermocouples



2 · Simulated vs measured T(t)



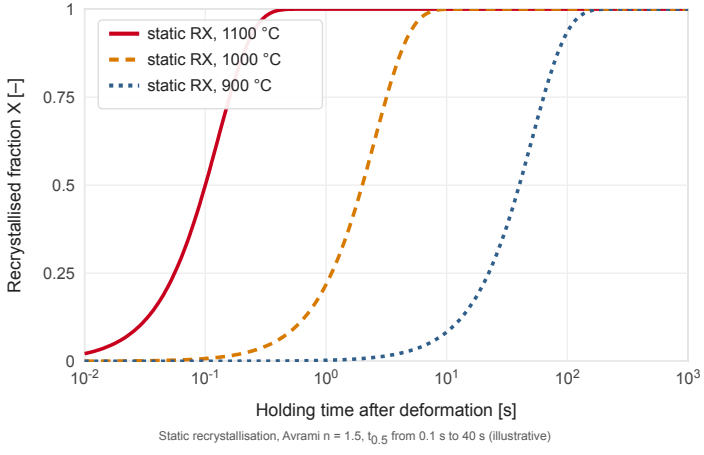
3 · Fitted HTC (© SFTC)

$$q = h\,(T_w - T_d), \quad \text{h = f(pressure, time, lubricant, scale)}$$

- Not a material property — set in **Inter-object**, but sits next to k in every heat balance
- DEFORM defaults: forming contact ≈ **11 N/(s·mm·°C)** (= 11 kW/(m²·K)), free surface convection ≈ 0.02

- **Inverse HTC**: DEFORM varies h(t) or h(T) until simulated thermocouple curves match the measured ones
- Same idea used for flow stress: **inverse analysis** of the compression test

Recrystallisation and grain size: JMAK (Avrami)



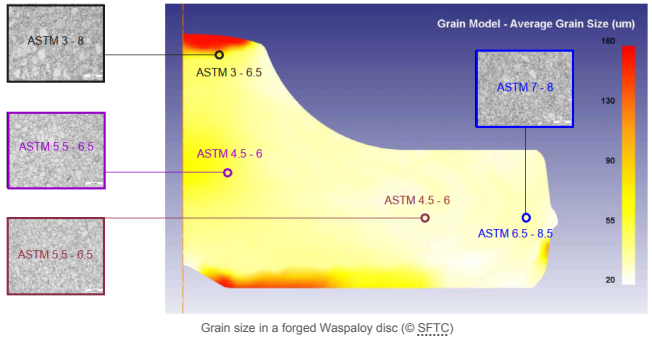
Static recrystallisation, Avrami n = 1.5, $t_{0.5}$ from 0.1 s to 40 s (illustrative)

$$X = 1 - \exp\left[-\beta \left(\frac{t}{t_{0.5}}\right)^n\right],\ \beta = \ln 2$$

$$t_{0.5} = a\, d_0^h\, \varepsilon^p\, \dot{\varepsilon}^q\, e^{Q/RT}$$

DRX grain: $d_{drx} = a\, d_0^h\, \varepsilon^n\, \dot{\varepsilon}^m\, e^{Q/RT}$

- DRX starts at $\varepsilon_c \approx 0.8\,\varepsilon_p$
- Data: Phase → **Grain** button



Grain size in a forged Waspaloy disc (© SFTC)

Phase transformation kinetics and hardness

Diffusional (ferrite, pearlite, bainite)

$$\xi = 1 - \exp(-b\,t^{n}), \quad \text{b, n from TTT + Scheil additivity}$$

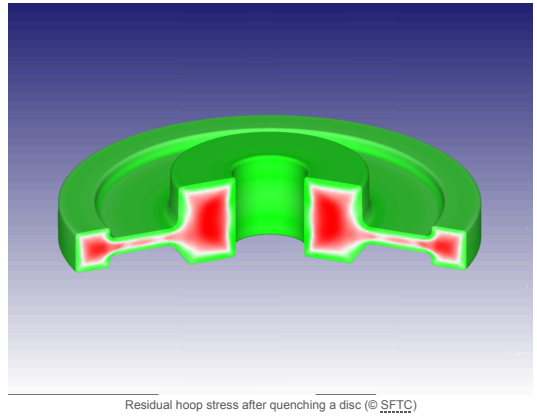
Martensite — Koistinen–Marburger

$$\xi_M = 1 - \exp[-\Omega\,(M_s - T)], \quad \Omega \approx 0.011\ \mathrm{K}^{-1}$$

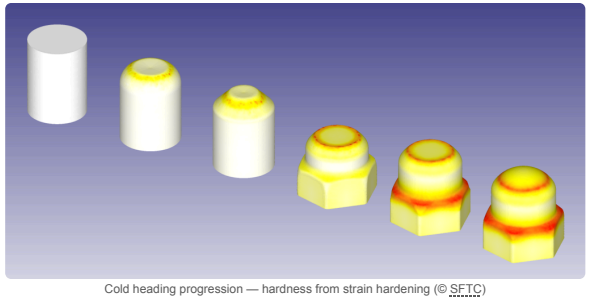
Hardness — Hardness tab

$$HV = \textstyle\sum_i \xi_i\, HV_i(\mathrm{chem.}, \dot{T})$$

- TTT / CCT from tests or from composition (Material Suite, JMatPro)



Residual hoop stress after quenching a disc (© SFTC)



Cold heading progression — hardness from strain hardening (© SFTC)

Diffusion (carburising) and electro-magnetic data (induction)

Diffusion tab

$$\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C), \quad D = D_0 \, e^{-Q/RT}$$

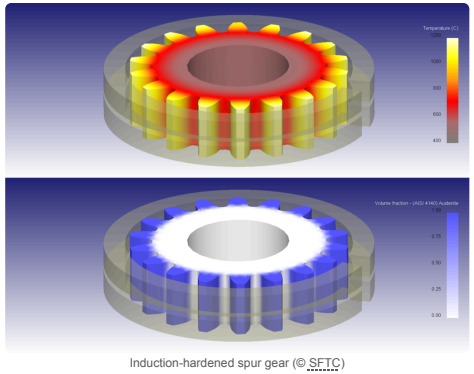
- Carbon in austenite: $D_0 \approx 2.3 \cdot 10^{-5}$ m²/s, $Q \approx 148$ kJ/mol
- Surface: carbon potential + mass-transfer coefficient
- Carbon content → M_s, hardness, TTT shift

Elec./Mag. tab

- Electrical resistivity $\rho_e(T)$
- Relative magnetic permeability $\mu_r(T, H)$ — drops to 1 above Curie point (≈ 770 °C)

Skin depth: $\delta = \sqrt{\frac{2\rho_e}{\omega \, \mu_0 \mu_r}}$

- Above Curie: δ grows ≈ 10× → heating zone moves inward

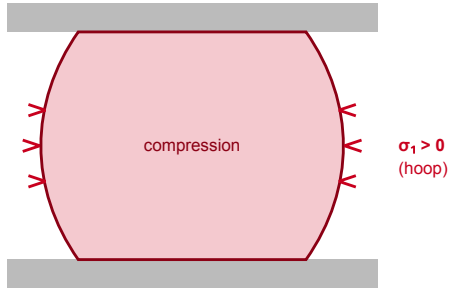


Induction-hardened spur gear (© SFTC)

Ductile fracture: normalised Cockcroft–Latham

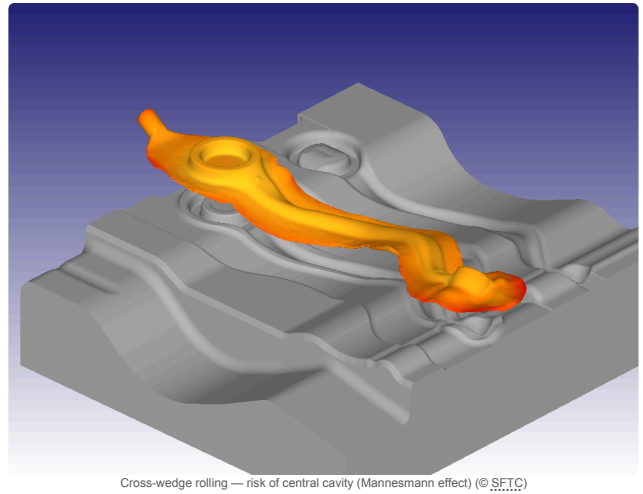
$$D = \int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \, d\bar{\varepsilon} \;\geq\; C$$

- Only **tensile** max. principal stress counts — compression does no damage
- C is a **material constant**, calibrated by simulating a test to its crack strain
- Typical C: 0.2 – 0.6 (cold steels) — depends on inclusions, T
- Advanced tab: damage model choice, element deletion threshold
- Others in DEFORM: Oyane, Rice–Tracey, Brozzo, McClintock, Lemaitre …



Free bulging surface: D reaches C first → surface cracks

Upsetting: tensile hoop stress at the bulge



Cross-wedge rolling — risk of central cavity (Mannesmann effect) (© SFTC)

Checklist: which material data does my simulation need?

Process	Flow stress	Yield / hardening	Elastic	Thermal	Extra
Cold forging, heading	Power law / table, ε to 2–3	von Mises, isotropic	For die stress	k, ρc_p (heating)	Damage C
Hot closed-die forging	Table σ(ε, ε̇, T) or sinh	von Mises, isotropic	–	**k(T), ρc_p(T), ε, HTC**	Grain (JMAK)
Machining	Johnson–Cook / ZA	von Mises	Tool	k, ρc_p	Damage, wear
Sheet / springback	Swift / Hollomon	**Hill 48 / Barlat, kinematic, Y-U**	**E($\bar{\varepsilon}^p$)**	–	FLD
Heat treatment	Per phase, f(T)	Isotropic (+ TRIP)	E(T), α(T)	Per phase	**TTT, latent heat, ΔV, HV**
Induction hardening	Per phase	Isotropic	E(T), α(T)	k, ρc_p	**ρ_e(T), μ_r(T)**

1. Check **units** (SI / English) · 2. Check **validity range** vs process · 3. Check **temperature dependence** · 4. **Validate** one result (force, T) against a measurement

Quick quiz & sources

1. Write the general flow-stress function and name its three variables.
2. Why can a rigid-plastic forging simulation run without E and v?
3. Which flow-stress model for machining, which for hot forging — and why?
4. What happens if the process exceeds the strain-rate range of a table?
5. How do R-values enter the Hill 48 criterion? What does Δr predict?
6. Isotropic vs kinematic hardening: what happens to the yield surface?
7. Heat capacity vs specific heat — what does DEFORM expect in SI units?
8. How is the Cockcroft–Latham constant C obtained?

- SFTC: DEFORM V14 manual, Ch. 10 Material Data; Material Suite; deform.com
- Necpal, M.; Sobota, R.: *Modelovanie a simulácia procesov tvárnenia*. STU, 2023
- Kobayashi, Oh, Altan: *Metal Forming and the FEM*. 1989
- Johnson & Cook (1983); Jaspers & Dautzenberg, *J. Mater. Process. Technol.* 122 (2002)
- Zerilli & Armstrong, *J. Appl. Phys.* 61 (1987)
- Sellars & Tegart (1966) — sinh law; Hill (1948); Barlat & Lian (1989), Barlat et al. (1991)
- Yoshida & Uemori, *Int. J. Plasticity* 18 (2002)
- EN 1993-1-2: thermal and mechanical properties of steel
- Images © SFTC, Gleeble/DSI, ZwickRoell, DoITPoMS — linked for teaching

NEXT LECTURE: PREPROCESSING — MESH, BOUNDARY CONDITIONS, FRICTION