T007: Strain Definitions and Calculations

Introduction

This document summarizes the theoretical definitions and calculations regarding strains, as they are used in DEFORM.

Problem Description

In DEFORM, the generalized elastic-plastic model with transformation can include several sources of strain. Pre-processing options found in 'Simulation Controls' cause some or all of these values to be stored in the database for viewing during post-processing. Theoretically, the sum of the components observed in the postprocessor should match the predicted total strain. However, for numerical reasons, the components and the total strain are each calculated by appropriate methods. Due to various numerical errors, the sums will not always be identical to the total. This article explains the strain calculation methods used in DEFORM.

Background Information

For an increment of deformation, the total strain is given by the theoretical description:

$$de_{ij}^{Total} = de_{ij}^{E} + de_{ij}^{p} + de_{ij}^{q} + de_{ij}^{Tr} + de_{ij}^{Tp}$$

where the subscripts indicate the following strain components:

- E – elastic strain
- P – plastic strain (plastic flow + creep)
- q – dilatational thermal strain
- Tr – dilatational transformation strain
- Tp – deviatoric strain due to the “transformation plasticity” effect

Solution

Strain Definitions

For this discussion, we will focus on incremental strains. The strain values reported in DEFORM are the sum of the strain increments.

Elastic Strain

Elastic strains are related to the applied stress by the elastic modulus (or Youngs modulus E) and poisson's ratio n.

$$\dot{\varepsilon}_{ij}^{E} = \frac{1+\nu}{E}\dot{\sigma}_{ij} - \frac{\nu}{E}$$

Elastic strains are fully recoverable when stress is removed.

Plastic Strain

Plastic strain represents permanent deformation in a part due to applied stress without transformation. In DEFORM, plastic strain can be governed either by the flow stress equations or by creep equations. In post-processing, plastic strain represents the portion governed by the flow stress law during deformation calculations. Creep represents only the part that was governed by the creep law during calculations.

Dilational Thermal Strain

Thermal strain results from temperature differences, and is recoverable upon returning to the original temperature.

$$\dot{\varepsilon}_{ij}^{T} = \alpha\left(T - T_0\right)\delta_{ij}$$

Where α is the thermal expansion coefficient, and δ is the Kronecker delta for tensor notation.

Dilational Transformation Strain

Transformation strains are strictly volumetric and are due to changes in specific volume when a material transforms from one phase to another. They are only active in simulations with transformation calculations enabled.

Transformation Plasticity

The transformation plasticity effect is a deviatoric strain. It represents a reduction in apparent flow stress of a material during transformation. It is only active when transformation calculations are enabled.

Strain Calculations

Thermal, transformation, and transformation plasticity strains are straightforward calculations.

For and elastic-plastic object elastic, plastic, and total strains are interrelated. In the absence of thermal, transformation or transformation plasticity strains, elastic and plastic strains should theoretically sum to total strain. However, enforcing this condition introduces other errors, so the values are computed independently.

Plastic strain

Elastic-plastic solutions use the “radial return” algorithm. The algorithm is complex, and a google search turns up several good explanations. For the sake of this discussion, it is sufficient to say that the solution uses an iterative method to determine the increment of elastic and plastic strain at each step. Total plastic strain is based on the sum of strain increments from each step.

Elastic strain

Using a sum of incremental elastic strain from the radial return calculations gives large errors. The elastic strain definition is simply the elastic stress divided by the elastic modulus.

Total strain

Total strain ultimately must account for not only elastic, and plastic, but also thermal and transformation strains. At each step, the total strain increment is based on standard gradient calculations from the displacement of the nodes, using the standard definition of strain found in any plasticity book. The total strain displayed in the postprocessor is the sum of total strain increments.

Related Information

For a more detailed description of strains, refer to a plasticity textbook such as MECHANICAL METALLURGY by George Dieter, McGraw Hill, 1986, Chapter 2.