Derivation of $\sigma = K\varepsilon^n$ curve from yield and UTS Data:

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The derivation is based on the observation that at the onset of plastic instability (necking) in a tensile test, the true strain is numerically equal to the strain hardening coefficient n. This strain corresponds to the Ultimate tensile stress (UTS) in a tension test.

Basic power law flow stress

$$\sigma = K \varepsilon^{n} \tag{1}$$

Taking the natural log of both sides of (1)

$$\ln(\sigma) = \ln(K) + n\ln(\varepsilon) \tag{2}$$

We have 2 unknowns (K,n), so we need 2 equations and 2 data points.

We know (From Kalpakjian) that at ε=n the engineering stress σ_E=UTS. (3)

We can assume that at a very small plastic strain (Say 0.002) $\sigma_E \cong$ YS (4)

Substituting (4) into (2) gives us

$$\ln(YS) = \ln(K) + n\ln(0.002) \tag{5}$$

Where, for small deformation, engineering and true strain are approximately equal.

At the UTS, we cannot make the small strain assumption, so we need to relate (3) to true stress and true strain

$$\sigma_T = \sigma_E (e+1) = \sigma_E(\exp(\varepsilon))$$
$$\sigma_{T-UTS} = UTS \exp(\varepsilon) \tag{6}$$

At a true strain ε= n, the true stress σ_T = UTS (exp(n)) (7)

Rearranging (5) gives

$$n = \frac{\ln(YS) - \ln(K)}{\ln(0.002)} \tag{8}$$

at UTS, ε = n.

$$\ln(\sigma) = \ln(k) + n\ln(\varepsilon)$$

(2)

Substitute (7) into (2)

$$\ln\left(UTS\exp(n)\right)=\ln\left(K\right)+n\ln\left(n\right) \tag{9}$$

Rearrange

$$\ln\left(K\right)+n\ln\left(n\right)-\ln\left(UTS\exp\left(n\right)\right)=0 \tag{10}$$

The solution is iterative. Guess a value for K. Calculate n using (8). Substitute values of K and n into (10). Iterate until the error in (10) = 0.